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An Optimization Method to Determine the Least Common Multiply

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Abstract. This paper proposes an Integer Programming model to obtain the Least Common Multiply (LCM) for some integer numbers. The proposed method is illustrated by a numerical example.

Keywords: Integer Programming (IP), Least Common Multiply (LCM), Number Theory (NT).

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1. Introduction

Linear integer programming refers to the class of combinatorial constrained optimization problems with integer variables, where the objective function and constrains are linear. The Integer Linear Programming (ILP) optimization problem can be stated in the following general form:

$$\begin{aligned} \max\ & cx \\ \text{s.t}\ \ & Ax \leqslant b, \\ & x \in \mathbb{Z}^n. \end{aligned}$$

where the solution $x \in \mathbb{Z}^n$ is a vector of n integer variables and the $x = (x_1, x_2, \ldots, x_n)^T$ data are rational and are given by the matrix $A_{m\times n}$, the $1 \times n$ matrix c, and the $m \times 1$ matrix b. This formulation includes also equality constraints,

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because each equality constraint can be represented by means of two inequality constraints.

A wide variety of real life problems in logistics, economics, social science and politics can be formulated as linear integer optimization problems. The combinatorial problems, such as the knapsack-capital budgeting problem, warehouse location problem and many scheduling problems can also be solved as linear integer optimization problems [6]. A Mixed Integer Programming (MIP) ("mixed" due to the fact that some of the variables are restricted to get only integer values) problem is an optimization problem with a linear objective function and linear constraints [7].

There are different methods in NT to determine the LCM. But, this paper uses the optimization method as a tool to determine the LCM. The reminder of the paper is organized as follows. Section 2 presents a preliminary of the NT. Section 3 presents the propose model to find the LCM and generalizes the proposed method. Section 4 illustrates the proposed method by a numerical example. We conclude this paper in section 5.

2. Preliminary

Integer numbers and their concepts are very attractive issues in mathematics. One of the attractive operations in NT is division. There exist many theorems about this subject. Some of them are as follows:

Theorem 2.1 *Suppose a and b are two integer numbers and $a > 0$, then there exist unique integer numbers like r and q such that*

$$b = aq + r, \ \ 0 \leqslant r < a. \tag{1}$$

Proof see [1]. ■

This theorem in NT so-called Division theorem.

Definition 2.2 *The number a divides the number b if and only if there exists an integer number like q such that $b = aq$. This relationship between a and b is demonstrated by symbol $a|b$. (Alternative terms are: a is divisor of b, or a is factor of b, or b is multiple of a). The symbol $a \not| b$ means that a does not divide b.*

2.1 *Least Common Multiply (LCM)*

Theorem 2.3 *Suppose a and b are two integer numbers and both of them are non-zero, then c is LCM of a and b if and only if:*

(1) $c > 0$,
(2) $a|c$ *and* $b|c$,
(3) *If* $a|e$ *and* $b|e$, *then* $c|e$.

The representation of LCM for a and b is $[a, b] = c$ [1]. In other words

$$[a, b] = \min\{c \in \mathbb{N} : \ a|c \ \& \ b|c\}.$$

Proof see [1]. ■

Theorem 2.4 *Suppose $[a, b] = c$ then*

(1) $[a, b] = [-a, b] = [a, -b] = [-a, -b] = c$,
(2) $[ma, mb] = |m|c$,
(3) $[a, b] = [b, a]$,
(4) $\max\{|a|, |b|\} \leqslant [a, b]$.

Proof see [1]. ■

Theorem 2.5 *The LCM always exists and is unique for every two or more numbers.*

Proof see [1]. ■

Definition 2.6 *(The LCM in general case)*
Suppose $a_1, \ldots, a_n$ *are* n *integers numbers and all of them are non-zero, then* c *is the LCM for* $a_1, \ldots, a_n$ *if and only if:*

(1) $c > 0$,
(2) $a_1|c, \ldots, a_n|c$,
(3) $a_1|e, \ldots, a_n|e$ *then* $c|e$.

The representation of LCM for $a_1, \ldots, a_n$ is: $c = [a_1, \ldots, a_n]$
In other words

$$[a_1, \ldots, a_n] = \min\{c : \ a_i|c \ for \ i = 1, \ldots, n\}.$$

3. The Propose Method for Computing LCM for *a* and *b*

Based on Definition 2, we must find the smallest c such that it satisfies in the condition of this definition. Hence, the corresponding optimization model is as follows:

$$\begin{aligned} &\min c \\ &\text{s.t.} \ \ a|c, \\ &\qquad b|c, \\ &\qquad c \in \mathbb{Z}. \end{aligned} \tag{1}$$

This model is an optimization problem but, we can obtain its integer linear programming form. By Definition 1, the final model is:

$$\begin{aligned} &\min c \\ &\text{s.t.} \ \ c = ak_1, \\ &\qquad c = bk_2, \\ &\qquad k_1 > 0, \ k_2 > 0, \ k_1, k_2, c \in \mathbb{Z}. \end{aligned} \tag{2}$$

Model (2) is an IP problem and its optimal value is the LCM.

3.1 *Computing the LCM in General Case by the Propose Method*

Based on Definition 4, we must find the smallest c such that it satisfies in the condition of this definition. Hence, the corresponding optimization model is as follows:

$$\begin{aligned} &\min c \\ &\text{s.t. } a_1|c, \\ &\quad\; a_2|c, \\ &\quad\; \vdots \\ &\quad\; a_n|c, \\ &\quad\; c \in \mathbb{Z}. \end{aligned} \tag{3}$$

This model is an optimization problem but, we can obtain its integer linear programming form. By Definition 1, the final model is:

$$\begin{aligned} &\min c \\ &\text{s.t. } c = a_1k_1, \\ &\quad\; c = a_2k_2, \\ &\quad\; \vdots \\ &\quad\; c = a_nk_n, \\ &\quad\; k_1 > 0, \ldots, k_n > 0,\; k_1, \ldots, k_n, c \in \mathbb{Z}. \end{aligned} \tag{4}$$

4. Numerical Example

To illustrated the propose model, we consider the computing LCM for 20 and 15. The corresponding model is as follows:

$$\begin{aligned} &\min c \\ &\text{s.t. } c = 15k_1, \\ &\quad\; c = 20k_2, \\ &\quad\; k_1 > 0,\; k_2 > 0,\; k_1, k_2, c \in \mathbb{Z}. \end{aligned} \tag{5}$$

This is a simple IP problem and by using the corresponding method the optimal solution and LCM are as follows:

$$(k_1^*, k_2^*, c^*) = (4, 3, 60) \text{ and LCM} = 60.$$

5. Conclusion

This paper used the optimization method to find the LCM for two integer numbers. Then the propose method is generalized to find the LCM for optional integer numbers.

Finally the propose method illustrated by a numerical example.

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