

## Using Free Disposal Hull Models in Supply Chain Management

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Received: 23 January 2013; Accepted: 26 June 2013.

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**Abstract.** To help improve customer relationship management (CRM), greater emphasis is given to the aspect of quality in the supply chain management and improve supply chain performance through is integrated business management and strategic partnerships. On the other hand, strategic supply chain management decisions are made at a company level that determine benefits and efficiencies of the supply chain and effective management of supply chains assists to product and delivery of a variety of products at low cost, high quality products, short lead times and services at the least cost. Hence, performance evaluation is of most importance for effective supply chain management. This paper gives a new application of DEA model, and FDH model to evaluate the supply chain. The basic motivation is to ensure that efficiency evaluations are effected from only observed performances.

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**Keywords:** Data envelopment analysis, Supply chain management, Free disposal hull, Performance evaluation.

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## 1. Introduction

In recent years the customers have the upper hand. Therefore, it is important to target the most profitable customers of a company. In order to remain globally competitive the companies have to reorganize their strategy and manage things differently. In today competitive business environment, major emphasis is effectively managing the supply chain [6]. Hence, extensive researches have been carried out in finding better method of managing the supply chain and optimizing its performance. The supply chain performance evaluation problem is one of the most comprehensive strategic decision problems that need to be considered for long-term

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efficient operation of the whole supply chain [2, 8]. In this highly competitive incentive market, the best strategy for winning and retaining business is buyers and suppliers to work together i.e. as the partner [3, 4]. Attributes of a good supplier is to deliver on time, to provide consistent quality, to give a good price, to provide a good service backup, to keep the buyer informed of progress and so on. Our approach is evaluating of the performance in buyer-supplier relationships through the measurement of intensity and effectiveness in a supply chain; we use to evaluate, tools such as Data Envelopment Analysis (DEA) and Free Disposal Hull (FDH). And, also identifies the benchmarking units for inefficient supply chains to improve their performance using the non-convex DEA model and FDH model.

## 2. The DEA FDH formulation

The nonparametric approaches known as FDH and DEA are based on the idea of enveloping the data under various assumptions on the technologies like free disposability, convexity or scale restrictions without imposing any uncertain parametric structure. These two mathematical programming techniques allow the measurement of the relative distance that an individual Decision Making Units lies away from this estimated frontier. The frontier defines the relationship between inputs and outputs by depicting graphically the maximum outputs obtainable from the given inputs. Nowadays, DEA has been utilized worldwide for measuring efficiencies of banks, supplier selection, electric utilities, supply chain and so forth. In addition, the FDH approach was initiated by [1]; it relies on only the assumption that  $PPS_{FDH}$  is freely disposable for the inputs and outputs and is appropriate for the efficiency measurement at the patient level. Therefore, we FDH continue to build on our performance evaluation of supply chain.

### 2.1 Method

Different from the CCR and the BCC models, the FDH model does not operate to a convexity assumption. Instead, this model has a discrete nature, i.e. the efficient reference point for an inefficient DMU is not chosen as a point on a continuous efficiency frontier, but among the existing DMUs. A performance of supply chain can be defined as the quantities information which helps a manager to lead the action towards an objective or which helps him to evaluate the results of an action [5, 7]. So, the performance evaluation of using Free Disposal Hull (FDH) models in Supply Chain Management can be seen as information which gives two possibilities to the manager: whether he changes objectives or he modifies presses. In particular nonparametric frontier methods such as DEA and FDH have been developed to the application across a wide range of competitive environment. Assume that there are  $n$  supply chains (SCs) each of them producing  $Q$  outputs by consuming  $P$  inputs so that B represents the buyer and S represents the supplier. More formally each  $SC_j$ , ( $j = 1, 2, \dots, n$ ) is denoted by  $x_{pj}$  ( $p = 1, 2, \dots, P$ ) and  $K$  outputs from this  $B_j$ ,  $i_{kj}$  ( $k = 1, 2, \dots, k$ ) and these  $K$  outputs become the inputs to the  $S_j$  and are referred to as intermediate products. The outputs from the  $S_j$  are denoted by  $y_{qj}$ , ( $q = 1, 2, \dots, Q$ ) in Figure 1.

At first, we use the non-convex model to evaluate the efficiency of the  $d^{th}$  ( $d = 1, 2, \dots, n$ ) buyer (the buyer in the  $d^{th}$  supply chain) by the following model:

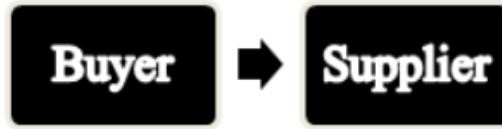


Figure 1. Buyer-Supplier process of supply chain

$$\begin{aligned}
 & \min \theta^{Bd} \\
 & \text{s.t.} \\
 & \sum_{j=1}^n \lambda_j^B x_{pj} \leq \theta^{Bd} x_{pd}, \quad p = 1, \dots, P, \\
 & \sum_{j=1}^n \lambda_j^B i_{kj} \geq i_{kd}, \quad k = 1, \dots, K, \\
 & \lambda_j^B \in \{0, 1\}, \quad j = 1, 2, \dots, n
 \end{aligned} \tag{1}$$

The non-convex efficiency of the  $d^{th}$  supplier is computed as follows:

$$\begin{aligned}
 & \max \varphi^{Sd} \\
 & \text{s.t.} \\
 & \sum_{j=1}^n \lambda_j^S i_{kj} \leq i_{kd}, \quad k = 1, \dots, K, \\
 & \sum_{j=1}^n \lambda_j^S y_{qj} \geq \varphi^{Sd} y_{qd}, \quad q = 1, \dots, Q, \\
 & \lambda_j^S \in \{0, 1\}, \quad j = 1, 2, \dots, n
 \end{aligned} \tag{2}$$

Therefore, the buyer-supplier of supply chain performance assessment model can be computed using model [3]:

$$\begin{aligned}
 & FDHSC_d = \min \theta^d \\
 & \text{s.t.} \\
 & \sum_{j=1}^n \mu_j x_{pj}^* \leq \theta^d x_{pd}, \quad p = 1, \dots, P, \\
 & \sum_{j=1}^n \mu_j y_{qj}^* \geq y_{qd}, \quad q = 1, \dots, Q, \\
 & \sum_{j=1}^n \eta_j x_{pj} \theta^{Bj^*} \leq x_{pd}^*, \quad p = 1, \dots, P, \\
 & \sum_{j=1}^n \eta_j i_{kj} = i_{kd}^*, \quad k = 1, \dots, K, \\
 & \sum_{j=1}^n \eta_j y_{qj} \varphi^{Sj^*} \geq y_{qd}^*, \quad q = 1, \dots, Q, \\
 & \mu_j^S \in \{0, 1\}, \quad j = 1, 2, \dots, n \\
 & \eta_j^S \in \{0, 1\}, \quad j = 1, 2, \dots, n
 \end{aligned} \tag{3}$$

Therefore, we can conclude;

1. FDHSC will have to demonstrate the success of cooperation,

2. FDHSC will have to clarify the strategic objectives of supply chain partners,
3. FDHSC will have to identify further potentials for improvement.

### 3. Application

This section examines the effect of joint decision making within 17 pairs of buyer-supplier relationships in manufacturing, Table1. Table 2 shows overall efficiency scores of supply chain, and reports the FDH efficiency scores of the two stages or subsystems.

Table 1. Data of 17 Supply Chain

NO.	$X_1$	$X_2$	$X_3$	$I_1$	$I_2$	$Y_1$	$Y_2$
SC1	1.0168	1.221	1.2215	166.9755	8.3098	0122.1954	3.7569
SC2	0.5915	0.611	0.4758	50.1164	1.7634	19.4829	0.66
SC3	0.7237	0.645	0.6061	48.2831	3.4098	34.412	0.7713
SC4	0.515	0.486	0.3763	35.0704	2.348	15.2804	0.3203
SC5	0.4775	0.526	0.3848	49.9174	5.4613	34.9897	0.8437
SC6	0.6125	0.407	0.3407	23.1052	1.2413	32.5778	0.4616
SC7	0.7911	0.708	0.4407	39.459	1.1485	30.2331	0.6732
SC8	1.2363	0.713	0.5547	37.4954	4.0825	20.6013	0.4864
SC9	0.446	0.443	0.3419	20.9846	0.6897	8.6332	0.1288
SC10	1.2481	0.638	0.4574	45.0508	1.7237	9.2354	0.3019
SC11	0.705	0.575	0.4036	38.1625	2.2492	12.0171	0.3138
SC12	0.6446	0.432	0.4012	30.1676	2.3354	13.813	0.3772
SC13	0.7239	0.51	0.37097	26.5391	1.3416	5.0961	0.1453
SC14	0.5538	0.442	0.3555	22.2093	0.9886	13.6085	0.3614
SC15	0.3363	0.322	0.2334	16.1235	0.4889	5.9803	0.0928
SC16	0.6678	0.423	0.3471	22.1848	1.1767	9.2348	0.2002
SC17	0.3418	0.256	0.15947	13.4364	0.4064	2.5326	0.0057

Table 2. The performance evaluation of 17 supply chain and identifies the benchmarking units.

NO.	$\theta^{Bj}$	$\varphi^{Sj}$	$\theta^j$	Benchmark
SC1	1	1	1	VSC1
SC2	1	1.02	0.8609	VSC5
SC3	0.8155	1	0.8155	VSC5
SC4	1.0000	1.4411	1.0000	VSC4
SC5	1.0000	1.0000	1.0000	VSC5
SC6	1.0000	1.0000	1.0000	VSC6
SC7	0.8732	1.0000	0.8732	VSC5
SC8	0.7377	1.0000	0.7377	VSC5
SC9	1.0000	1.0000	1.0000	VSC9
SC10	0.8413	2.2299	0.7449	VSC6
SC11	0.9534	1.4892	0.8688	VSC6
SC12	1.0000	1.2238	0.9502	VSC6
SC13	1.0000	3.1769	0.9186	VSC6
SC14	1.0000	1.0000	1.0000	VSC14
SC15	1.0000	1.0000	1.0000	VSC15
SC16	1.0000	1.0000	1.0000	VSC16
SC17	1.0000	1.0000	1.0000	VSC17

Our Table 2 results are indicative in many methods. The First, while the higher efficiency trend of the SC1, SC2, SC4, SC5, Sc6, SC9, SC12, and SC17 over the SC3, SC7, SC8 and SC11 and the second, our research proposed model directly identifies the benchmarking units for inefficient supply chains to improve their performance also.

#### 4. Conclusions

Member of supply chain and supply chain management try to improve its performance, but it is very hard to build good performance evaluation system. Of course, the performance evaluation system is different in each supply chain, but we employ data envelopment analysis for performance evaluation of two-stage supply chain in this paper. The current paper develops new supply chain DEA models using FDH aimed at (i) correctly via characterizing multi-member supply chain operations, and (ii) calculating the efficiencies of the supply chain and its members. The implications for the management of supply chain relationships and directly identify the benchmarking units for inefficient supply chains to improve their performance. Also, this method can be applied to supply chain with complicated construction that have mainly structure networking.

#### Acknowledgments

This study was supported in part by Islamic Azad University, Firoozkooch Branch, Firoozkooch, Iran. Thanks also go to the anonymous reviewers for their valuable comments and suggestions of this article.

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