

A Boundary-Fitted Shallow Water Model to Simulate Tide and Surge for the Head Bay of Bengal – Application to Cyclone Sidr(2007) and Aila(2009)

Farzana Hussain

*Department of Mathematics, Shahjalal University of Science & Technology, Sylhet-3114,
Bangladesh.*

Abstract. Severe Tropical Cyclones associated with surges frequently hits the coastal region of Bangladesh. For a reliable hydrodynamic model to simulate the severity of such cyclones, it is necessary to incorporate the meteorological and hydrological inputs properly. In order to incorporate the coastlines and the island boundaries properly in the numerical scheme a very fine grid resolution along the coastal belts is necessary. Consideration of very fine resolution involves more memory, CPU time in the solution process and invites problem of complexity or numerical instability in the model. In this study we use the boundary-fitted curvilinear grids where the complete boundary of the analysis area and each island are represented by four curves and they are defined by four functions. Using appropriate transformations of independent coordinates, the curvilinear physical domains are transformed to a square one and also each island boundary transforms to a rectangle within this square domain. The vertically integrated shallow water equations are transformed to the new space domain and then the regular explicit finite difference scheme is used to solve the shallow water equations, where the problem domain is divided into 100×129 grid points. The model is applied to compute the water levels due to astronomical tide and surges associated with Cyclone SIDR(2007) and Cyclone AILA(2009) that hit the coast of the Bay of Bengal. The computed results along the coastal belt of Bangladesh are found to be in good agreement with the observed water levels.

Received: 12 February 2013; Revised: 22 June 2013; Accepted: 1 July 2013.

Keywords: Cyclone SIDR, Cyclone AILA, Tropical Storms, Surge, Shallow water equations, Transformation of Coordinates, Bay of Bengal.

Index to information contained in this paper

- 1. Introduction**
- 2. Mathematical Formulation of the Problem**
- 3. Results and Discussions**
- 4. Conclusion**

*Corresponding author. Email: farzanahn@live.com&farzanahsn@gmail.com.

1. Introduction

Severe tropical storm along with surge is the most destructive natural disaster which frequently hits the coastal region of Bangladesh. On average, five to six storm form in this region every year. The associated surge is more dangerous rather than the storm itself. Sometimes it may rise from 9 to 15 meters (Roy [18]) and rushes towards the land, which causes severe damage to the life and property. Because of its complex coastal geometry, Bangladesh suffers more than the surrounding countries. The Bay of Bengal is surrounded by the coasts from all sides except in the south where there is open sea. The coastal geometry is curvilinear in nature and the bending of the coastline is very high moreover, there are many small and big islands in the offshore region of the Bangladesh coast. Various factors significantly increase the surge levels along the coast of Bangladesh such as: shallowness of water, off-shore islands, bending of coastlines, oceanic bathymetry, low lying islands, huge discharge through the rivers etc. Also, the head Bay of Bengal is a large tidal range area. Worst devastation may take place, if a storm approaches the coast at the time of high tide. Figure 1 shows the path of the cyclones and the cyclone affected areas of coastal belt of Bangladesh.

Different procedures and techniques for numerical model development are studied for the Bay of Bengal [3 - 7, 9 - 13]. The representation of the coastal boundaries in those models was not so accurate and none of the off-shore islands are included in those models because very fine resolution could not be considered. In a stair step model the curvilinear coastal and island boundaries are approximated along the nearest finite difference gridlines. Thus, if the grid size is not small, representation of the coastal and island boundaries are not accurate in those models. Also, a very fine grid resolution near the coast and offshore region is necessary to incorporate the island boundaries and the coastline properly through stair steps, which is not necessary away from the coast. Consideration of very fine resolution involves more computer memory and CPU time in the solution process and invites problem of numerical instability or complexity. In hydrodynamic models for coastal seas, bays, and estuaries the use of boundary-fitted curvilinear grids not only makes the model grids fit to the coastline, but also make the finite difference scheme simple and more accurate. In a boundary-fitted transformed coordinate model the curvilinear boundaries are transformed into straight ones, so that in the transformed domain regular finite difference technique can be used. Johns et. al.[10] first used the partially boundary-fitted curvilinear grids in their transformed coordinate model for the east coast of India. The works of Dubeet. al. [6], Johns et. al. [12, 13], Sinhaet. al. [21] were also based on representing the coasts by curvilinear boundaries and transformation of coordinates. But in those works two opposite boundaries were considered straight lines and none of them incorporated any offshore islands. The main difficulty in incorporating the islands was that, the whereabouts of the island boundaries were undetectable in the transformed domain. Roy [19] developed a model where a fine mesh numerical scheme for the Meghna estuary was nested into a coarse mesh scheme extending up to 15⁰ N lat. In the fine mesh scheme all the major islands were incorporated through proper stair step representation. That model was similar to Johns et.al. [13] for the East Coast of Indiawhere two opposite boundaries were considered straight lines. Recently [14-16] developed some models using nested numerical schemes. In [14] a fine mesh scheme incorporating the

coastal belt and islands has been nested in to a coarse mesh scheme covering up to 15° N latitude in the Bay of Bengal. In the fine mesh scheme all the major islands are incorporated through proper stair step representation. Again, a very fine mesh scheme for the region between Barishal and Chittagong is nested in to the fine mesh scheme. The model was tested for the storms of November 1970 and April 1990. [15] Simulated water levels for storm “AILA-2009” using nested numerical scheme. Nested numerical scheme was used in [16] to simulated water levels for storm “SIDR -2007”. The complexity of the nested numerical models lies in the matching of the boundary line of the fine mesh and coarse mesh scheme within the domain. ‘Boundary-fitted grid’ techniques for coastal and estuarine dynamics were developed for many other regions [1, 2, 11, 22-24].

The present study attempts to develop an accurate surge forecasting model based on transformation of coordinate. In the present study the complete boundary of the physical domain is represented by four boundary-fitted curves or functions. Based on them, the four boundaries of islands are approximated by two generalized functions. Using mathematical transformations the physical domain becomes a square one and each island became a rectangle in the transformed domain. The vertically integrated shallow water equations are transformed into the new domain the regular explicit finite difference scheme with 100 × 129 grid points with time step 30s is used to solve the shallow water equations. In this model the major islands Bhola, Hatiya, Sandwip are incorporated. The model is applied to compute the water levels due to tide and surge associated with the storm SIDR (2007) and AILA (2009) along the Bay of Bengal. Cyclone SIDR and AILA attacked the coast of Bangladesh, the impact of these storm was severe - people died and extensive damage occurred to life and property along the coastal region. For the analysis and verification of the model, results are taken at 8 locations along the coast. The locations are Hiron point, Barishal, Bhola (Island), Charjabbar (Noakhali), Hatiya (Island), Sandwip (Island), Chittagong and Cox’s Bazar. It is found that the model is quite capable to compute the tide and surge along the coast of Bangladesh and the results are found reasonably accurate.

2. Mathematical Formulation of the Problem

2.1 Boundary-Fitted Grids

A system of rectangular Cartesian coordinate is used in which the origin, O , is the mean sea level (average level of sea surface). OX , OY , OZ directed towards the south, the east and vertically upwards respectively. The northern and southern coastal boundary of Bangladesh, is given by $x = \beta_1(y)$ and $x = \beta_2(y)$ respectively. The western and eastern coastal boundaries are at $y = \delta_1(x)$ and $y = \delta_2(x)$ respectively. This configuration is shown in Figure 2. It may be seen in the figure that the southern boundary $x = \beta_2(y)$ is taken as a straight line but this may be considered as a curve as well. Also it is to be noted that, the functions are not defined by explicit expressions, rather they are defined in tabular form. The boundary-fitted grids are generated through the following generalized functions: The system of gridlines along $x = \beta_1(y)$ and $x = \beta_2(y)$ are given by the generalized function

$$x = \{(m - q) \beta_1(y) + q \beta_2(y)\} / m \quad (1)$$

where m and q are constants and $0 \leq q \leq m$.

The system of gridlines along $y = \delta_1(x)$ and $y = \delta_2(x)$ are given by the generalized function

$$y = \{(l - p) \delta_1(x) + p \delta_2(x)\} / l \tag{2}$$

where l and p are constants and $0 \leq p \leq l$.

We note that, (1) reduces to $x = \beta_1(y)$ and $x = \beta_2(y)$ for $q = 0$ and $q = m$ respectively. Similarly, (2) reduces to $y = \delta_1(x)$ and $y = \delta_2(x)$ for $p = 0$ and $p = l$ respectively. By proper choice of q, m and p, l we can generate the boundary-fitted curvilinear grid lines.

2.2 Coordinate Transformation

A coordinate transformations based upon a new set of independent variables ξ, η , and t given by

$$\xi = \frac{x - \beta_1(y)}{\beta(y)}, \quad \beta(y) = \beta_2(y) - \beta_1(y) \tag{3}$$

$$\eta = \frac{y - \delta_1(x)}{\delta(x)}, \quad \delta(x) = \delta_2(x) - \delta_1(x) \tag{4}$$

These transform the physical curvilinear domain into the following rectangular one

$$0 \leq \xi \leq 1, \quad 0 \leq \eta \leq 1$$

Also, the generalized functions given by (1) and (2) transforms to

$$\xi = q / m \tag{5}$$

$$\eta = p / l \tag{6}$$

The coastal boundary $x = \beta_1(y)$ or $\xi = 0$ are obtained for $q = 0$ and the open sea boundary $x = \beta_2(y)$ or $\xi = 1$ are obtained for $q = m$. Similarly, for $p = 0$, we have the western coastal boundary $y = \delta_1(x)$ or $\eta = 0$ and for $p = l$, we have the eastern coastal boundary $y = \delta_2(x)$ or $\eta = 1$. The appropriate choice of the constants m & l and the parameters q & p in (5) and (6) will generate the rectangular grid system in the transformed domain. Curvilinear boundaries of typical domain and the curvilinear grid system are shown in Fig. 3a. It may be noted that one of the boundaries is taken as straight line. In fact, it can be curved line as well. The corresponding boundaries and the rectangular grid system after the transformation are shown in Fig 3b.

2.3 Representation of Islands

The northern and southern boundaries of an island are given by (1) and the western and eastern boundaries are given by (2). Using the transformations (3) and (4) the boundaries of the island are given by (5) and (6). Thus two different values of q , say, q_1 and q_2 with $q_1 < q_2$ the northern and southern boundary of an island can be expressed by (5). Similarly, two different values of p , say, p_1 and p_2 with $p_1 < p_2$ the western and eastern boundary of the island will expressed by (6). Thus the transformed boundaries of an island are expressed as

$$\xi = \frac{q_1}{m}, \quad \xi = \frac{q_2}{m}, \quad \eta = \frac{p_1}{l}, \quad \eta = \frac{p_2}{l} \tag{7}$$

2.4 Vertically Integrated Shallow Water Equations

The displaced position of the sea surface and the position of the sea floor are considered as $z = \zeta(x, y, t)$ and $z = -h(x, y)$ respectively. The vertically integrated shallow water equations given by Roy [19] are

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} \{(\zeta + h)u\} + \frac{\partial}{\partial y} \{(\zeta + h)v\} = 0 \tag{8}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv = -g \frac{\partial \zeta}{\partial x} + \frac{\tau_x}{\{\rho(\zeta + h)\}} - \frac{c_f u(u^2 + v^2)^{1/2}}{(\zeta + h)} \tag{9}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu = -g \frac{\partial \zeta}{\partial y} + \frac{\tau_y}{\{\rho(\zeta + h)\}} - \frac{c_f v(u^2 + v^2)^{1/2}}{(\zeta + h)} \tag{10}$$

The wind field over the physical domain is derived from the empirical formula given by Jelesnianski [8]

$$\begin{aligned} V_a &= V_o(r/R)^{3/2} \quad \text{for } r \leq R \\ &= V_o(R/r)^{1/2} \quad \text{for } r > R \end{aligned} \tag{11}$$

Radial and tangential components of wind stress are derived by

$$(\tau_r, \tau_\theta) = C_D \rho_a (u_a^2 + v_a^2)^{1/2} (u_a, v_a) \tag{12}$$

τ_x and τ_y , the x and y component of wind stress in (9) and (10) are derived from τ_r and τ_θ .

2.5 The Boundary Conditions

The boundary conditions used in this model following Johns et. al. [13] are given by

$$u - v \frac{d}{dy}(\beta_1) = 0 \quad \text{at } x = \beta_1(y) \tag{13}$$

$$u - v \frac{d}{dy}(\beta_2) = (g/h)^{1/2} \zeta \quad \text{at } x = \beta_2(y) \tag{14a}$$

$$v - u \frac{d}{dx}(\delta_1) = 0 \quad \text{at } y = \delta_1(x) \tag{15}$$

$$v - u \frac{d}{dx}(\delta_2) = 0 \quad \text{at } y = \delta_2(x) \tag{16}$$

For generating tide in the basin the southern open sea boundary condition is taken as

$$u - v \frac{d}{dy}(\beta_2) = (g/h)^{1/2} \zeta - 2(g/h)^{1/2} a \text{Sin}[(2\pi)/T + \varphi] \quad \text{at } x = \beta_2(y) \tag{14b}$$

where a and φ denote respectively the prescribed amplitude and phase of the tidal force and T is the tidal period.

2.6 Governing Equations and Boundary Conditions in the Transformed Domain

For the transformation (3) and (4), we have

$$\frac{\partial}{\partial x} \equiv \frac{\partial \xi}{\partial x} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x} \frac{\partial}{\partial \eta}$$

$$\frac{\partial}{\partial y} \equiv \frac{\partial \xi}{\partial y} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial y} \frac{\partial}{\partial \eta}$$

Using these operators in (8) – (10), we have the following transformed equations:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial \xi} \{(\zeta + h)U\} + \frac{\partial}{\partial \eta} \{(\zeta + h)V\} = 0 \tag{17}$$

$$\begin{aligned} \frac{\partial u}{\partial t} + U \frac{\partial u}{\partial \xi} + V \frac{\partial u}{\partial \eta} - f v = -g \left[\frac{\partial \zeta}{\partial \xi} \frac{1}{\beta} - \frac{\partial \zeta}{\partial \eta} \frac{\{(\delta_1)_x + \eta \delta_x\}}{\delta} \right] \\ + \frac{\tau_x}{\rho(\zeta + h)} - \frac{c_f u (u^2 + v^2)^{1/2}}{\zeta + h} \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{\partial v}{\partial t} + U \frac{\partial v}{\partial \xi} + V \frac{\partial v}{\partial \eta} + f u = -g \left[\frac{\partial \zeta}{\partial \eta} \frac{1}{\delta} - \frac{\partial \zeta}{\partial \xi} \frac{\{(\beta_1)_y + \xi \beta_y\}}{\beta} \right] \\ + \frac{\tau_y}{\rho(\zeta + h)} - \frac{c_f v (u^2 + v^2)^{1/2}}{\zeta + h} \end{aligned} \tag{19}$$

where

$$U = u \frac{\partial \xi}{\partial x} + v \frac{\partial \xi}{\partial y} = \frac{u - \{(\beta_1)_y + \xi \beta_y\}v}{\beta} \tag{20}$$

$$V = u \frac{\partial \eta}{\partial x} + v \frac{\partial \eta}{\partial y} = \frac{v - \{(\delta_1)_x + \eta \delta_x\}u}{\delta} \tag{21}$$

The boundary condition (13) - (16) transform to

$$U = 0 \quad \text{at } \xi = 0 \tag{22}$$

$$\beta U - (g/h)^{1/2} \zeta = 0 \quad \text{at } \xi = 1 \tag{23a}$$

$$\beta U - (g/h)^{1/2} \zeta = -2(g/h)^{1/2} a \text{Sin}[(2\pi t)/T + \varphi] \quad \text{at } \xi = 1 \tag{23b}$$

$$V = 0 \quad \text{at } \eta = 0 \tag{24}$$

$$V = 0 \quad \text{at } \eta = 1 \tag{25}$$

The normal component of velocity vanishes at each boundary of an island. Thus from (7), the boundary conditions for an island are given by

$$U = 0 \quad \text{at } \xi = q_1/l \quad \& \quad \xi = q_2/l \tag{26}$$

$$V = 0 \quad \text{at } \eta = p_1/l \quad \& \quad \eta = p_2/l \tag{27}$$

2.7 Numerical Set up of the Model

The curvilinear grid system in the physical domain is generated through (1) and (2). In the transformed domain the corresponding rectangular grid system is generated through (5) and (6) with appropriate choice of m , l , q , and p . The curvilinear grid system is shown in Fig 3a and the corresponding rectangular grid system is shown in Fig 3b.

Let us define discrete coordinate points in the transformed domain by

$$\xi = \xi_i = (i - 1) \Delta \xi, \quad i = 1, 2, \dots, n_i$$

$$\eta = \eta_j = (j - 1) \Delta \eta, \quad j = 1, 2, \dots, n_j$$

A sequence of time instant is given by

$$t = t_k = k\Delta t, k = 1, 2, 3, \dots$$

In the computational domain we use staggered grid in which there are three distinct types of computational points. With i even and j even, the point is a ζ - point at which ζ is computed. If i is odd and j is even, the point is a u - point at which u is computed. If i is even and j is odd, the point is a v - point at which v is computed. We choose $n_i (= 100)$ to be even so that at the southern open boundary there are ζ - points and v - points only. Similarly we choose $n_j (= 129)$ to be odd thus ensuring that there are only ζ - points and v - points at the eastern and western boundaries. The coastal boundary is approximated either along the nearest odd grid line ($i = \text{odd}$) given by (1) so that we have only u - points on this part of the boundary or along the nearest odd grid line ($j = \text{odd}$) given by (2) so that we have only v - points along that part of the boundary. The island boundaries are also approximated in the same manner. Thus, the boundaries of the coast and of the islands are represented by such a system of stair steps that, at each segment (stair) there exists only that component of velocity which is normal to the segment. This is done in order to ensure the vanishing of the normal component of velocity at the boundaries in the numerical scheme.

The governing equations (17) – (19) and the boundary conditions (22) – (27) are discretized by finite difference (forward in time and central in space) and are solved by conditionally stable semi- implicit method using staggered grid as mentioned earlier. For numerical stability, the velocity components in (18) and (19) are modeled in a semi-implicit manner. For example, in the last term of (18) the time discretization of $\tilde{u}(u^2 + v^2)$ is done as $\tilde{u}^{k+1}(u^2 + v^2)^k$ where the superscript k and $k+1$ denote values at the present and advanced time levels respectively. Moreover, the CFL criterion has been followed in order to ensure the stability of the numerical scheme. Along the closed boundary, the normal component of the velocity is considered as zero, and this is easily achieved through appropriate stair step representation as mentioned earlier.

The initial value of ζ , u , and v are taken as zero. The time step is taken as 30s that ensures stability of the numerical scheme. In the solution process, a uniform value of 0.0026 for the friction coefficient (C_f) and 0.0028 for the drag coefficient (C_D) are considered throughout the physical domain.

In this model the analysis area is extended from 84° E to 96° E along the coast of Bangladesh, India, and Myanmar. The open sea boundary is situated along 18° N (Fig. 2). The east-west extent of the analysis area varies between 734 km and 1035 km and the north-south extent varies between 208 km and 541 km. The analysis area has been divided to 100 x 129 grid points. Thus in the north-south direction Δx varies between 2.08 km and 5.41 km while in the east-west direction Δy varies between 5.734 km and 8.085 km. In the transformed domain we consider $\Delta\xi = 1.0/(n_i - 1)$, and $\Delta\eta = 1.0/(n_j - 1)$ so that $q_i = (i - 1)\Delta\xi = \xi_i$ and $p_j = (j - 1)\Delta\eta = \eta_j$.

The offshore region of Bangladesh coast is full of big and small islands with a high density around the Meghna estuary. In this study it is possible to incorporate the small islands by considering very fine resolution in the numerical scheme; we consider the major islands Bhola, Hatiya, Sandwip more accurately (Fig. 2).

3. Results and Discussions

3.1 Analysis of the Computed Surge Response

The model is applied to compute the water levels due to tide and surge associated with a few tropical storms that hit the coast of Bangladesh. To analyze the result we have chosen the storms of November 2007 (SIDR) and May 2009 (AILA) with maximum sustained anti-clock wise circulatory wind velocities of 250 km/h (69.44 m/s) and 120 km/h (33.33 m/s) respectively. Table.1 gives the history of the above-mentioned storms, the data of which was received from the Bangladesh Meteorological Department (BMD) (Figure 1a and 1b).

Cyclone ‘SIDR’ was one of the 10 strongest cyclones that hit Bangladesh between 1876 and 2007 and is also favorable for high surge due to both wind intensity and path of its movement. So we give more stress on the 2007 storm data for computation and verification of results. It was formed on November 11, dissipated on November 16, 2007. It intensified into a category 4 storm with the peak sustained wind of 250 km/h. A catastrophic surge flooded the coastal area and caused most of the deaths and the damages. SIDR officially made landfall around 17:00 UTC on 15 November with sustained winds of 215 km/h. The hardest-hit area was Barguna (Barishal) located near 22.9N latitude and 89.2E longitude. According to Bangladesh Meteorological Department (BMD) the entire cities of Patuakhali, Barguna and Jhalokati District (Barishal division) were hit hard by the storm surge of about 5 meters (16 ft). Figure 4a, b depicts the computed surge levels associated with SIDR at different coastal locations. It may be observed in the figures that, the surge level is increasing with time as the storm approaches towards the coast and finally there is recession. At Hiron Point a strong recession is occurred after 14:00 UTC of 15th November, earlier than in any other locations and about 3 hrs before the landfall time. The recession takes place due to backwash of water from the shore towards the sea. In fact, Hiron Point is situated along left (west) of the storm path and so the direction of the anti-clock wise circulatory wind becomes northerly (i.e. towards the sea) at Hiron Point before the storm reaches the coast and thus driving the water towards the sea. The recession reaches up to -1.9 m approximately at 16:00 UTC of 15th November. It may be noticed that the recession at Kuakata, Bhola, Hatiya, Char Jabbar, Chittagong and Cox’s Bazar began later than Hironpoint (Figs. 4). Thus the beginning of recession delays as we proceed towards east as is expected. At every location, the peak surge is attaining before the land falls time of the storm. This is expected, as the circulatory wind intensity is highest along the coast when the storm reaches near the coast. According to storm surge analysis by the Institute of Water Modeling (IWM), there was 4 to 6 meters surge along the coastal regions between Hiron point and Kuakata. Thus, the computed surge heights are almost identical with the report. The same information is also found in the website of Wikipedia [26] and NASA [28].

According to Bangladesh meteorological Department (BMD) Cyclone AILA was formed on May 23, dissipated on May 26, 2009. On the 21st of May 2009, the Joint Typhoon Warning Center (JTWC) reported that a Tropical Disturbance had persisted about 950

kilometers to the south of Kolkata, in India. The Depression continued to slowly intensify until it was upgraded to a Deep Depression and designated as Tropical Cyclone 02B by the JTWC. Later, the deep depression had intensified into a Cyclonic storm and had been named as 'AILA'. AILA became a severe cyclonic storm at 06:00 UTC on May 25. The peak sustained wind was 120 km/h. AILA officially made landfall between 08:00 and 09:00 UTC of 25 May with peak sustained winds of 110-120 km/h along the west of border area of Bangladesh and India. Figure 5a, b depicts the computed surge levels associated AILA at different coastal locations. It may be observed that, the maximum surge level is increasing with time as the storm approaches towards the coast and finally there is recession. The recession (after landfall) is very low, because the entire Bangladesh coast is situated along the right of the storm path and so the direction of the anti-clock wise circulatory wind becomes northerly (i.e. towards the land) at every location and thus driving the water was towards the land, not towards the sea. The locations Hatiya, Sandwip, Chittagong and Cox's Bazar are situated far left of the cyclone, so surge levels at those locations are comparatively lower. According to storm surge analysis by the Institute of Water Modeling (IWM) Bangladesh, there was 3 to 6 meters surge along the coastal regions between Hiron point and Charjabbar of which 2 to 4 meter is due to the astronomical tide, because AILA made landfall at the time of high tidal period. The computed surge heights are almost identical with the report of IWM and the website of Wikipedia [27] and NASA [29].

Experiment is done to test the sensitivity of surge level with respect to the intensity of a storm. Figure 6 shows the peak surges along the coastal locations due to the storms of 2007 and 2009 respectively. The surge due to November 2007 storm is found to be much higher, which not only because of strong wind but also because of a favorable path for generating high surge (Roy, [18]). It may be noticed that, surge level at locations along the left side of the figure is higher for 2009 storm though its wind velocity is less than that of 2007. This is attributed to the path of the storm (Table 1). The locations along the left side of the figure are situated along the immediate left side of the landfall point of AILA. It may be observed that, the coastal region between Barisal (Kuakata) and Chittagong is vulnerable for very high surge.

3.2 Analysis of the Computed Tide and Tide-Surge Interaction

We know, the astronomical tide is a continuous process in the sea, the surge due to tropical storms always interacts with the astronomical tide. So the pure tidal oscillation is the initial dynamical condition for interaction of tide and surge. A way of incorporating tidal oscillation with surge is to superimpose linearly the time series of surge response obtained through model simulation with that of oscillation obtain from tide table. The tidal information is generally available, as high and low values, four times a day in Bangladesh Tide Table.

The tide is generated in the model through the south open boundary condition (23b) with appropriate values of a , T , and ϕ in absence of wind stress. It is observed that, though there is variation in the tidal period in the head Bay of Bengal, the average period is approximately of M_2 tide and so we choose $T = 12.4$ hrs. By trial it is found that $\phi = 0$ is a good choice for head Bay region. The information of the amplitude a along the southern

boundary is not available. We have chosen $a = 0.6 \text{ m}$ to test the response of the model along the coastal belt (Figure 7). It is found that response is exactly sinusoidal with the same period (12.4 hrs), which is expected. About the phase it may be seen that Hatiya, Sandwip, Chittagong and Cox's Bazar are in the same phase of tidal oscillation. This is because of the fact that, they are very close to each other. Hence providing appropriate values of the amplitude a along the southern open sea boundary condition the model can be used to generate the actual tidal oscillation in the whole basin.

Figure 8 shows the computed tide, computed surge, and their linear interaction associated with May 2009 storm at Hiron Point, Kuakata, Bhola and Charjabbar. According to storm surge analysis by the Institute of Water Modeling (IWM), there was 3 to 6 meters surge along the coastal regions between Hiron point and Charjabbar of which 2 to 4 meter is due to the astronomical tide, because AILA made landfall at the time of high tidal period. Thus, the computed water levels are almost identical with the report of IWM. The storm approaches the coast during high tide period and hence intensifies the water level due to interaction. At each location, because of weak wind the surge response is less when the storm is away from the coast and the total water level is dominated by tidal oscillation. On the other hand, because of very strong wind the water level is dominated by surge when the storm approaches the coast. The maximum water level occurred at Hironpoint is 3 m, at Kuakata is approximately 3.3 m, at Bhola is 4 m and at Charjabbar is 5 m.

High tide is at approximately 1 a.m. local time and SIDR made landfall about halfway between low and high tide. For this reason we are neglecting the tide and surge interaction for 2007 storm (SIDR) results. Thus, the storm tide could have been about 2-3 feet higher than the observed water levels, since there is a 1.5 meter (5 feet) difference between low and high tide in western Bangladesh coast.

3.3 Comparison Between Computed and Observed Time Series of Water Levels

The verification of a model is dependent on the correct observational data. But time series of observed authentic water level data are very limited. We could not compare our computed results to the observed data due to non availability of observed time series data. The Hydrographic department of BIWTA collects water level data at different coastal locations through manual gauge readers. During a severe storm period it is not possible to stay in the gauge station to collect the data, so observed time series data are not available. The results of the article are explained and compared with the information obtained from the Institute of Water Modeling (IWM), Bangladesh Inland Water Transport Authority (BIWTA), Bangladesh Meteorological department (BMD) and the website of Wikipedia and NASA.

Experiment is done to test the influence of offshore islands over the surge amplitude along the coastal belt. Figure 9 depicts the maximum surge levels at different coastal locations due to 2007 storm in the presence of island and in the absence of island. The computed results show that in the presence of islands the surge level decreases. Finally, Figure 10 shows the Contours (in meters) of equal sea surface elevation for peak surge along the Bay of Bengal for the storms SIDR (2007) and AILA (2009). It is found that the region between Barishal (Kuakata) and Cox's Bazar is vulnerable for high surge, which is in agreement with the observation.

4. Conclusion

In this study we have developed a numerical model based on transformation of coordinates to compute the tide and surge levels associated with a storm along the coast of Bangladesh. This shallow water model can be applied to compute water levels along the East Coast of India and the coast of Myanmar also. The model is also applicable for any bay or estuary or even in any confined lake. The model is capable of incorporating the bending of coastline and island boundaries with a considerable accuracy. The model is tested for 30×65 , 60×65 , 90×91 grid points with suitable time steps (results not shown) and the model results are found to be in good agreement with existing methods and observed results. Also it is found that the region between Barishal (Kuakata) and Cox's Bazar is vulnerable for high surge for the coastal region of Bangladesh, which is also in agreement with the observation.

References

- [1] Androsov, A. A., Vol'tzinger, N. E, Liberman, Y. M.: A two dimensional tidal model of the Barents Sea; *Oceanology*, **37** (1997) 16 – 22.
- [2] Bao, X.W., Yan, J., Sun, W. X.: A three-dimensional tidal model in boundary-fitted curvilinear grids; *Estuarine, Coastal and Shelf Science*, **50** (2000) 775 – 788.
- [3] Das, P. K.: Prediction model for storm surges in the Bay of Bengal; *Nature*, **239**(1972) 211- 213.
- [4] Das, P. K., Sinha M. C., Balasubrahmanyam, V.: Storm surges in the Bay of Bengal; *Quart J. Roy. Met. Soc.*, **100** (1974) 437- 449.
- [5] Dube, S. K., Sinha, P. C., Roy, G. D.: The numerical simulation of storm surges along the Bangladesh coast; *Dyn. Atmos. Oceans*, **9** (1985) 121-133.
- [6] Dube S. K., Sinha, P. C., Roy, and G. D.: The numerical simulation of storm surges in Bangladesh using a bay-river coupled model; *Coastal Engineering*, **10** (1986) 85-101.
- [7] Flierl, G. R., Robinson, A. R.: Deadly surges in the Bay of Bengal: Dynamics and storm tide tables; *Nature*, **239** (1972) 213-215.
- [8] Jelesnianski, C. P.: A numerical calculation of storm tides induced by a tropical storm impinging on a continental shelf; *Mon. Wea. Rev.*, **93** (1965) 343-358.
- [9] Johns B., Ali, A.: The numerical modeling of storm surges in the Bay of Bengal; *Quart. J. Roy. Met. Soc.* **106** (1980) 1-18.
- [10] Johns B. Dube, S. K. Mohanti, U. C., Sinha, P. C.: Numerical simulation of surge generated by the 1977 Andhra cyclone; *Quart. J. Roy. Soc. London*, **107** (1981) 919-934.
- [11] Johnson, B. H.: Numerical modeling of estuarine hydrodynamics on a boundary fitted coordinate system; *Numerical Grid Generation* (Thompson, J., ed.) Elsevier, (1982) 419 – 436.

- [12] Johns B., Dube, S. K., Mohanti, U. C., Sinha, P. C.: The simulation of continuously deforming lateral boundary in problems involving the shallow water equations; *Computers and Fluids*,**10** (1982) 105-116.
- [13] Johns, B., Rao, A. D., Dube, S. K., Sinha P. C.: Numerical modeling of tide-surges interaction in the Bay of Bengal; *Phil. Trans. R. Soc. of London* *A313* (1985) 507-535.
- [14] M. M. Rahman, A. Hoque, G. C. Paul and M. J. Alam: Nested Numerical Schemes to Incorporate Bending Coastline and Islands of Bangladesh and Prediction of Water Levels due to Surge; *Asian Journal of Mathematics and Statics*, **4(1)** (2011) 21 -32.
- [15] M. M. Rahman, A. Hoque, G. C. Paul: A Shallow Water Model Water Model for the coast of Bangladesh and Applied to Estimate water levels for 'AILA'; *Journal of Applied Sciences*, **11 (24)** (2011) 3821 -3829.
- [16] M. Mizanur Rahman, G. Chandra Paul and A. Hoque,,: A Cyclone Induced Storm Surge forecasting Model for the coast of Bangladesh with Application to the Cyclone 'Sidr'; *International Journal of Mathematical Modelling & Computations*, **01 (02)** (2011) 77 -86.
- [17] Roy, G. D.: Some aspects of storm surges along the coast of Bangladesh; *Ganith (J. Bangladesh Math. Soc.)*,**6(1)** (1985) 1- 8.
- [18] Roy, G. D.: Estimation of expected maximum possible water level along the Meghna estuary using a tide and surge interaction model; *Environment International*,**21(5)** (1995) 671- 677.
- [19] Roy G. D.: Inclusion of offshore islands in transformed coordinates shallow water model along the coast of Bangladesh; *Environment international*,**25(1)** (1999) 67- 74.
- [20] Roy G. D., HumayunKabir, A. B. M., Mandal, M. M., Haque, M. Z.: Polar coordinate shallow water storm surge model for the coast of Bangladesh; *Dyn. Atms. Oceans*, **29** (1999) 397- 413.
- [21] Sinha, P. C., S. K. Dube, and G. D. Roy: Numerical simulation of storm surges in Bangladesh using a multi-level model; *Int. J. for Num. methods in fluids*,**6** (1986) 305-311.
- [22] Sheng, Y. P.: On modelling three- dimensional estuarine and marine hydrodynamics; *Three- dimensional Models of Marine and Estuarine Dynamics*(Nihoul J. C.J, ed.) Elsevier Oceanography Series,**45** (1989) 35 – 54.
- [23] Spaulding, M. L.: A vertical averaged circulation model using boundary-fitted coordinates; *Journal of Physical Oceanography*,**14** (1984) 973 – 982.
- [24] Thompson, J.F., Frank, C., Thames, F.C., Mastin, C.W. Automatic numerical generation of body-fitted curvilinear systems for tied containing any number of arbitrary two-dimensional bodies; *Journal of Computational Physics*, **15** (1974) 229-319.
- [25] Hussain Farzana, A Transformed Coordinates Shallow water Model for the Head of the Bay of Bengal using Boundary-Fitted Curvilinear Grids, *East Asian Journal on Applied Mathematics*, **3(1)** (2013) 27-47.
- [26] Wikipedia web site - http://en.wikipedia.org/wiki/Cyclone_Sidr
- [27] Wikipedia web site - http://en.wikipedia.org/wiki/Cyclone_Aila

- [28] Website -
http://www.nasa.gov/mission_pages/hurricanes/archives/2007/h2007_sidr.html
 [29] Website
 -http://www.nasa.gov/mission_pages/hurricanes/archives/2009/h2009_aila.html

Table 1. History of the chosen storms

<u>Storm of 2007 (Sidr)</u>				<u>Storm of 2009 (Aila)</u>			
Date	Hour	Lat.	Long.	Date	Hour	Lat.	Long.
1112	0600	10.8	90.4	0524	0000	18.1	88.5
1112	1200	11.0	90.3	0524	0600	18.4	88.4
1112	1800	11.6	90.0	0524	1200	18.3	88.4
1113	0000	12.1	89.9	0524	1800	19.1	88.2
1113	0600	12.5	89.8	0525	0000	20.3	88.1
1113	1200	13.0	89.6	0525	0600	21.6	88.3
1113	1800	13.7	89.5	0525	1200	22.9	88.2
1114	0000	14.3	89.6	0525	1800	24.2	88.5
1114	0600	15.0	89.4	0526	0000	24.2	88.5
1114	1200	15.7	89.3	0526	0600	24.2	88.5
1114	1800	16.6	89.3	0526	1200	27.1	89.2
1115	0000	17.8	89.2	0526	1800	27.1	89.2
1115	0600	19.3	89.3				
1115	1200	20.9	89.5				
1115	1800	22.8	90.3				
1116	0000	25.0	91.9				

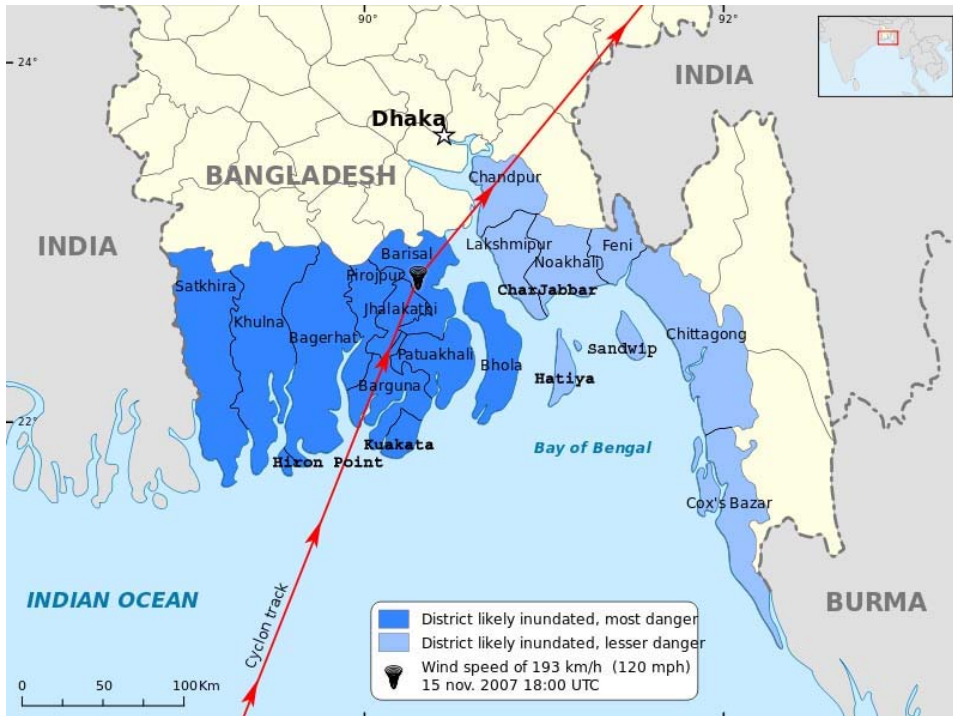


Figure 1a. Path of Cyclone Sidr (2007) and affected area with the analysis and the locations at which results are presented. Source: Bangladesh Space Research and Remote Sensing Organization.



Figure 1b. Path of Cyclone Aila (2009) and affected area.

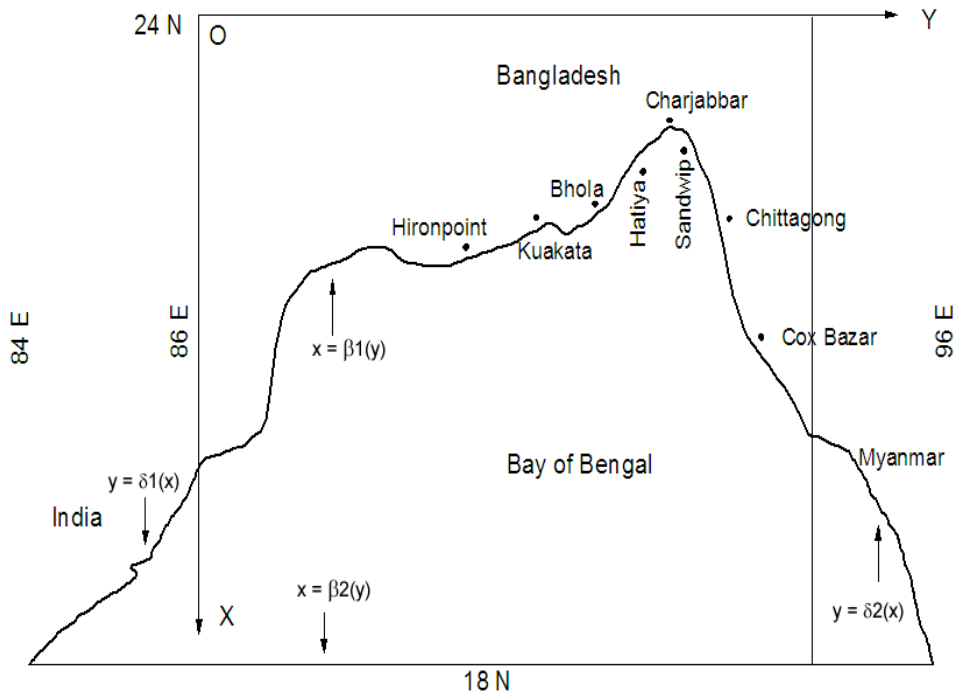


Figure 2. Boundaries of the analysis area and the locations.

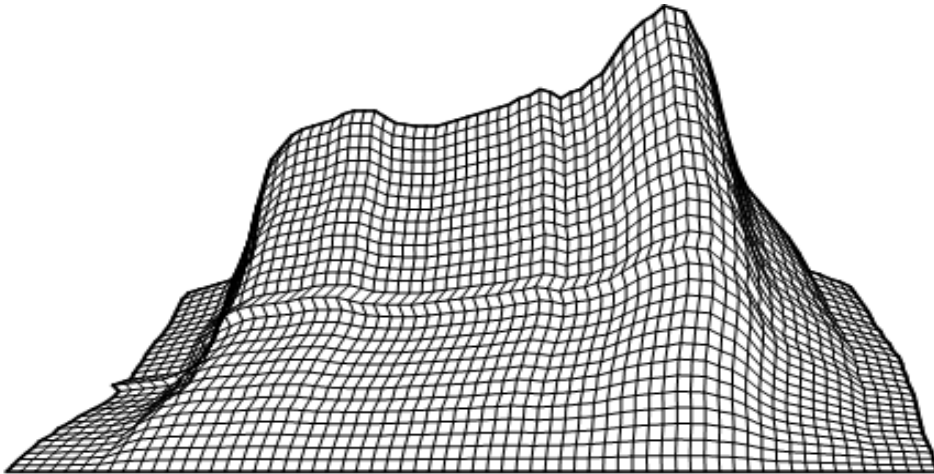


Figure 3a. Curvilinear boundaries and the curvilinear grid system.

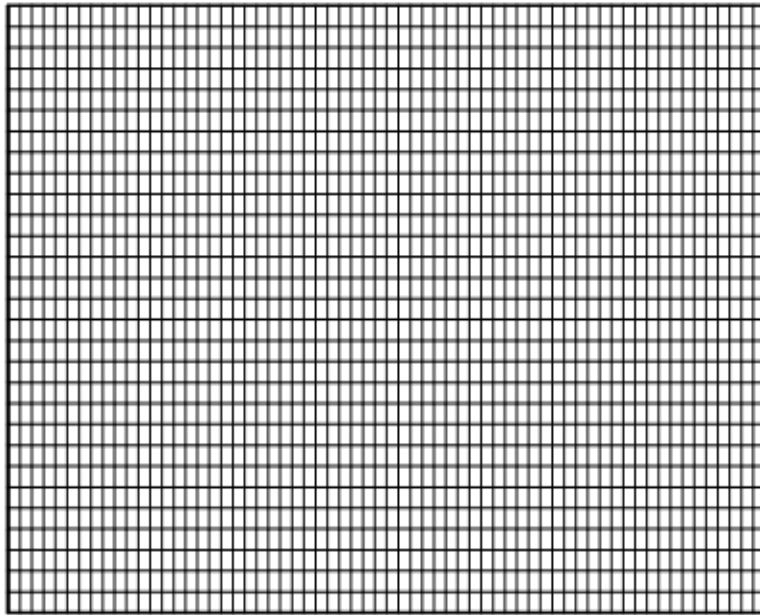


Figure 3b. Boundaries and rectangular grid system in the transformed domain.

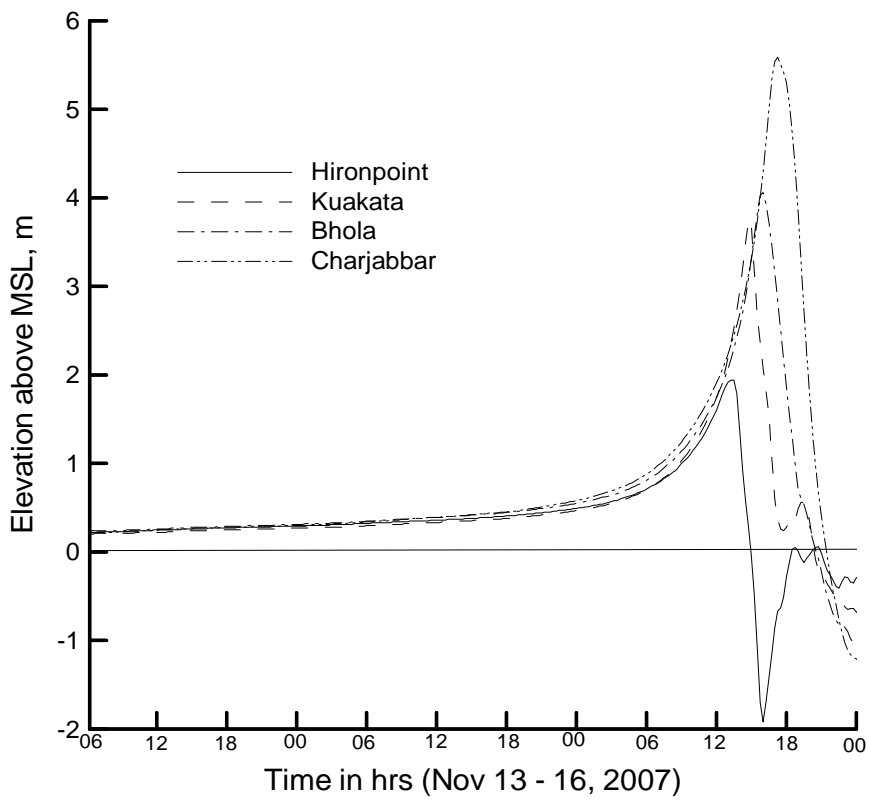


Figure 4a. Computed time series of surge levels at the coastal locations associated with November 2007 storm (Sidr).

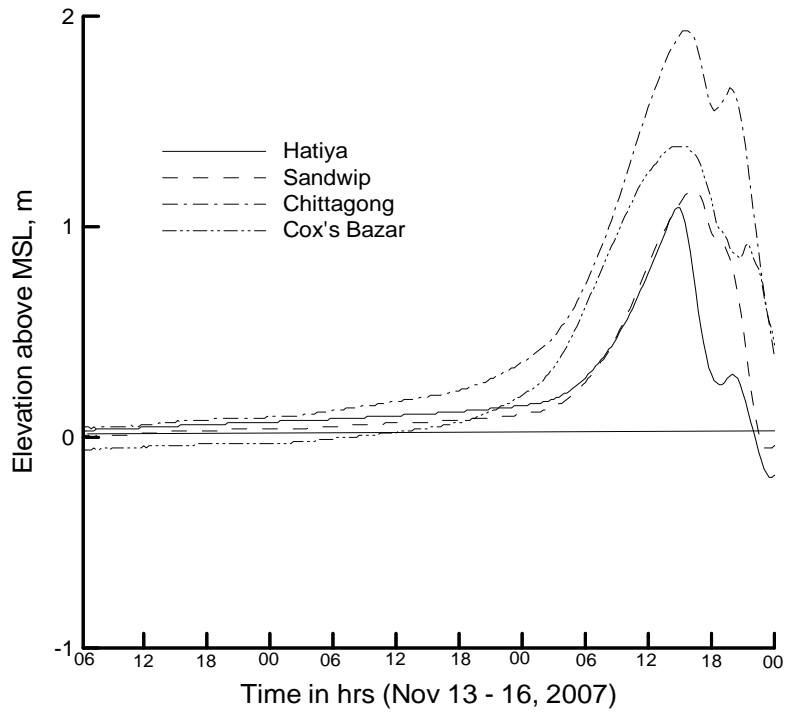


Figure 4b. Computed time series of surge levels at the coastal locations associated with November 2007 storm (Sidr).

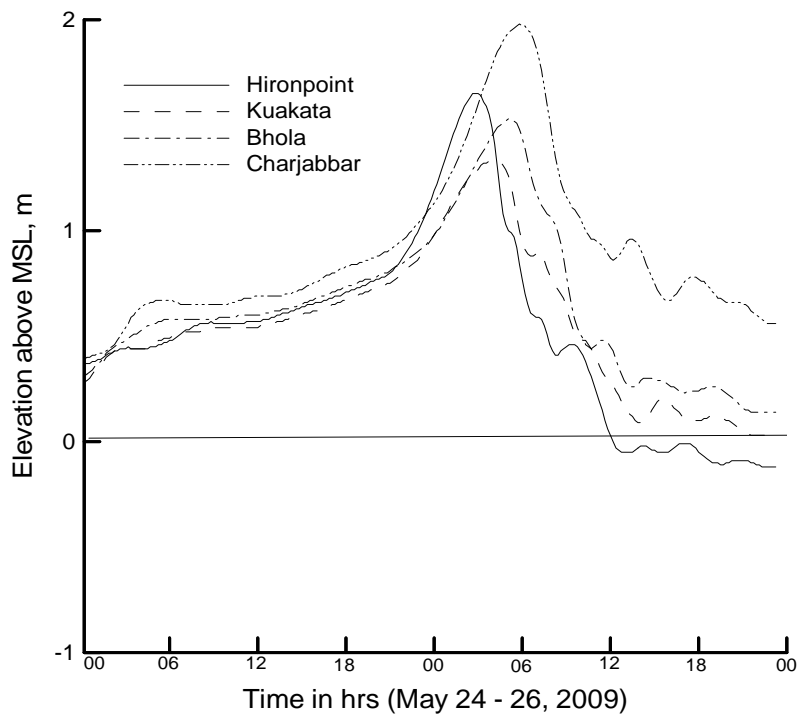


Figure 5a. Computed time series of surge levels at the coastal locations associated with May 2009 storm (Aila).

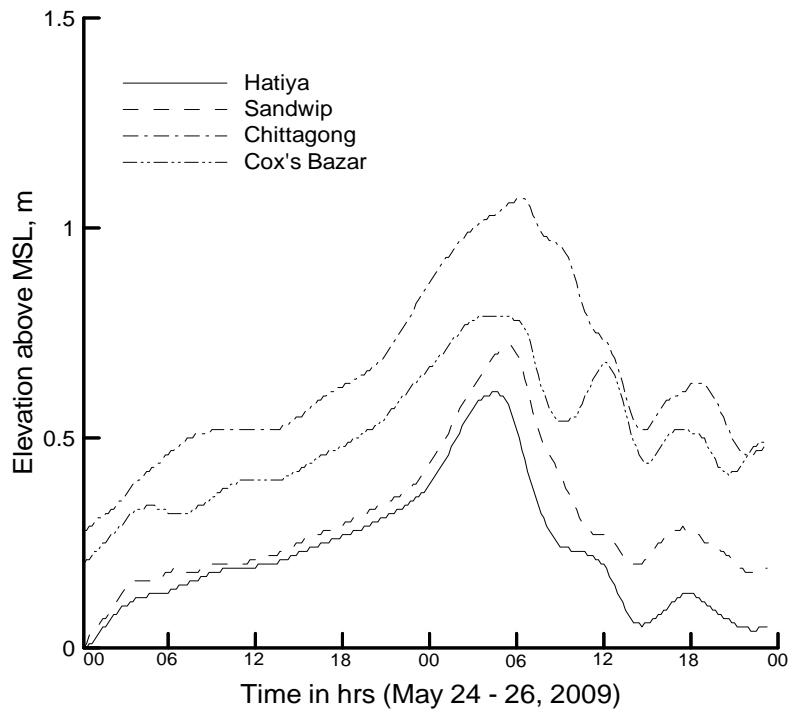


Figure 5b. Computed time series of surge levels at the coastal locations associated with May 2009 storm (Aila).

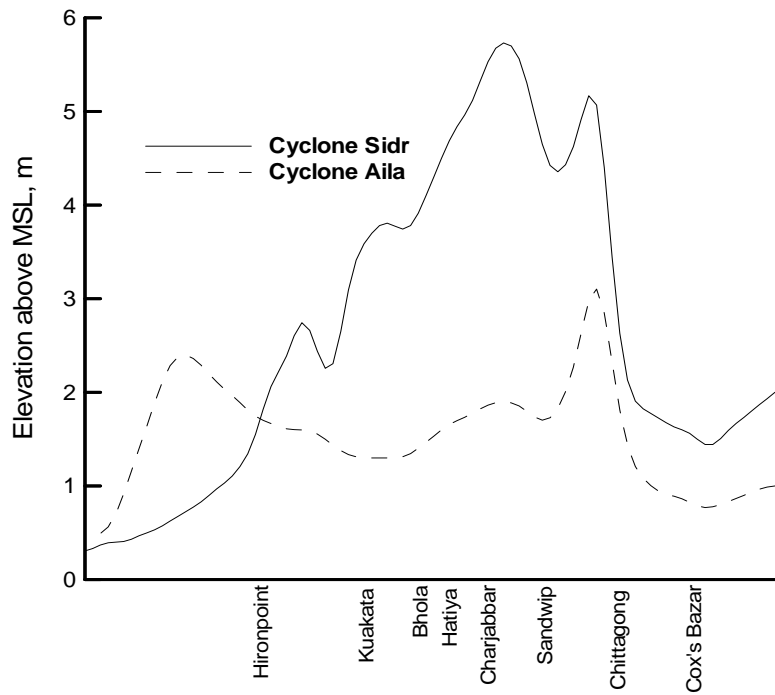


Figure 6. Peak surges along the coastal locations due to the storms of 2007 (Sidr) and 2009 (Aila).

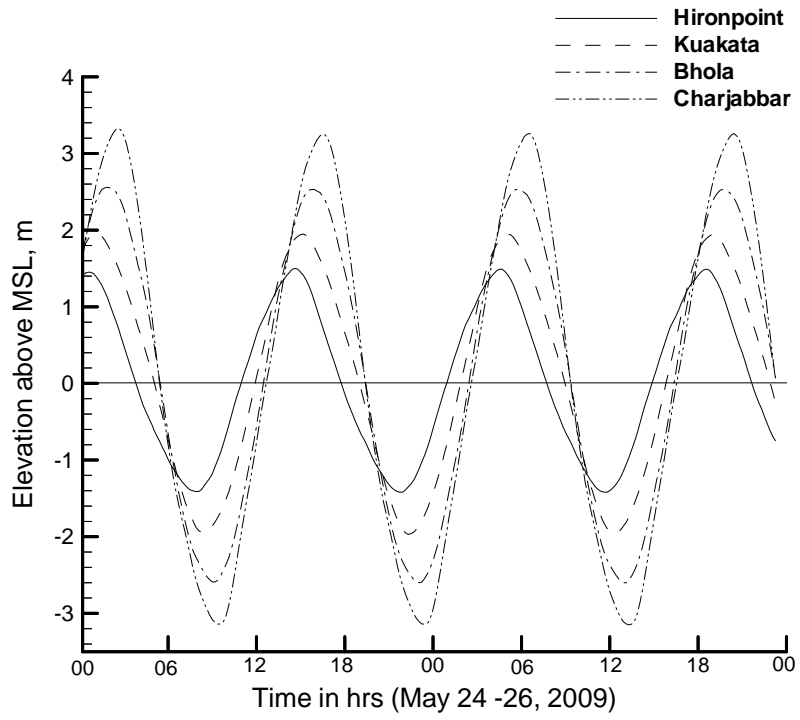


Figure 7a. Computed tidal oscillations at different coastal locations.

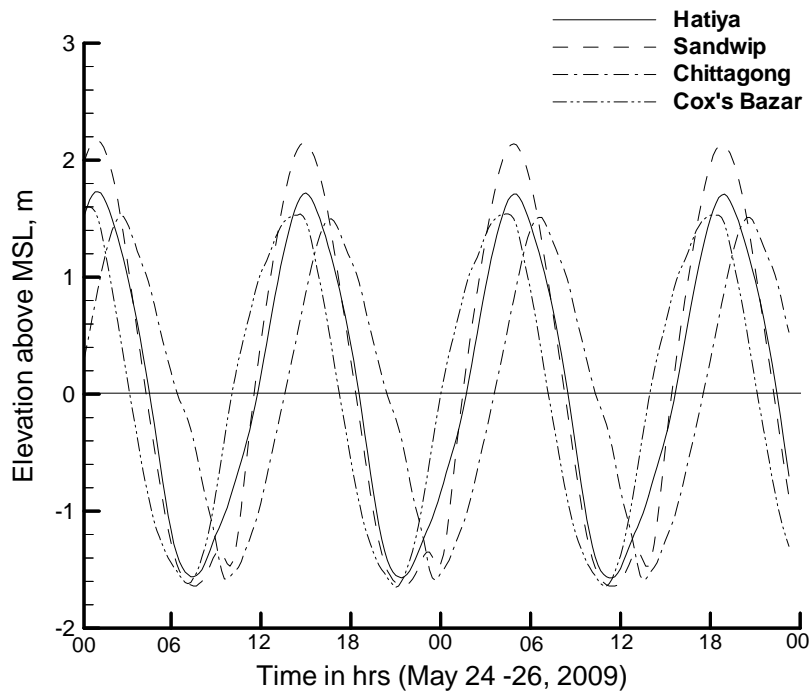


Figure 7b. Computed tidal oscillations at different coastal locations.

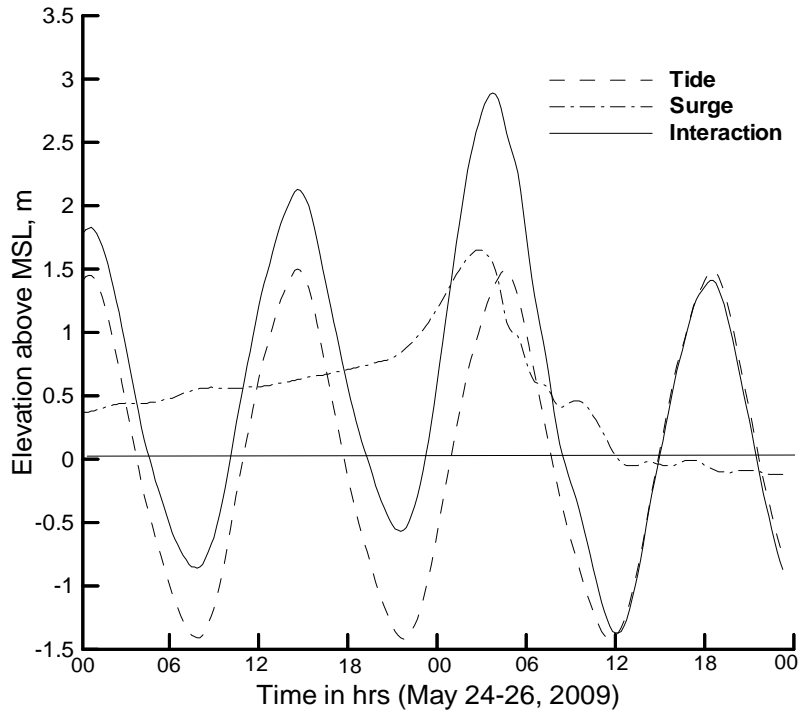


Figure 8a. Computed tide, computed surge, and their linear interaction associated with May 2009 storm at Hiron Point.

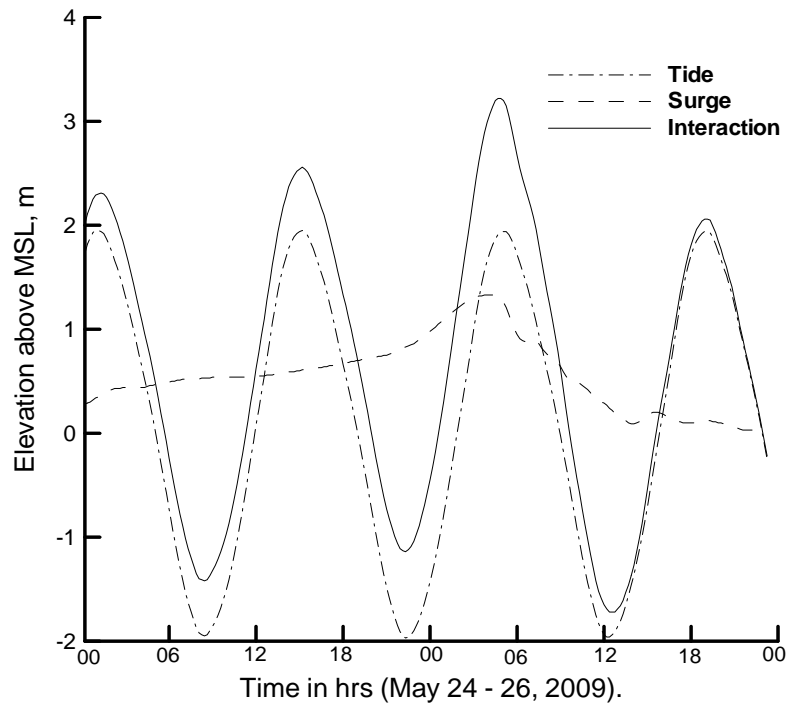


Figure 8b. Computed tide, computed surge, and their linear interaction associated with May 2009 storm at Kuakata.

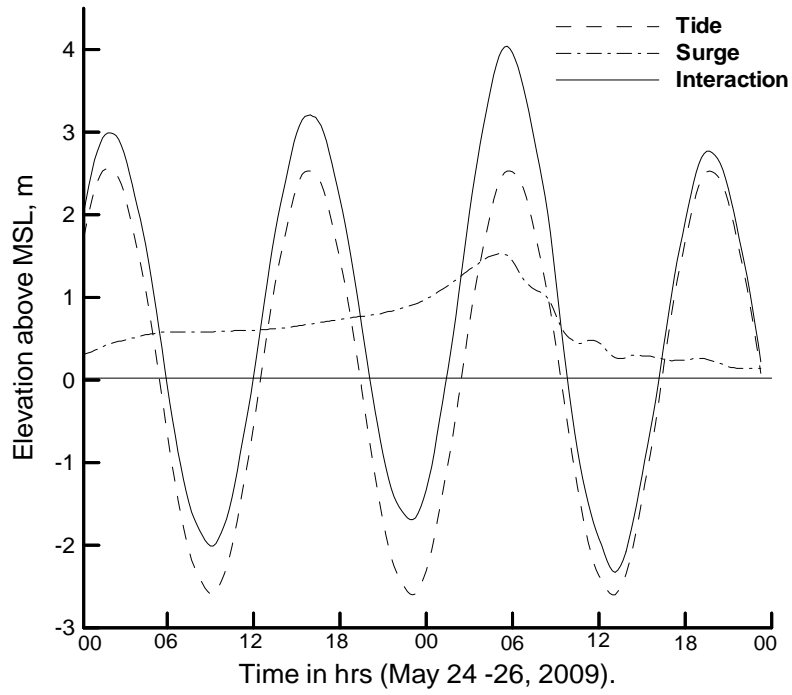


Figure 8c. Computed tide, computed surge, and their linear interaction associated with May 2009 storm at Bhola.

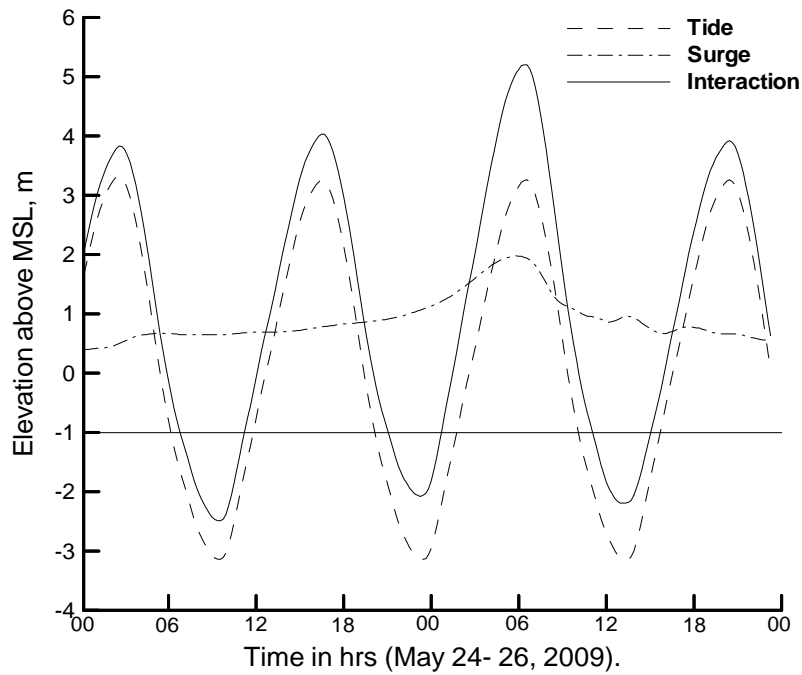


Figure 8d. Computed tide, computed surge, and their linear interaction associated with May 2009 storm at Charjabbar.

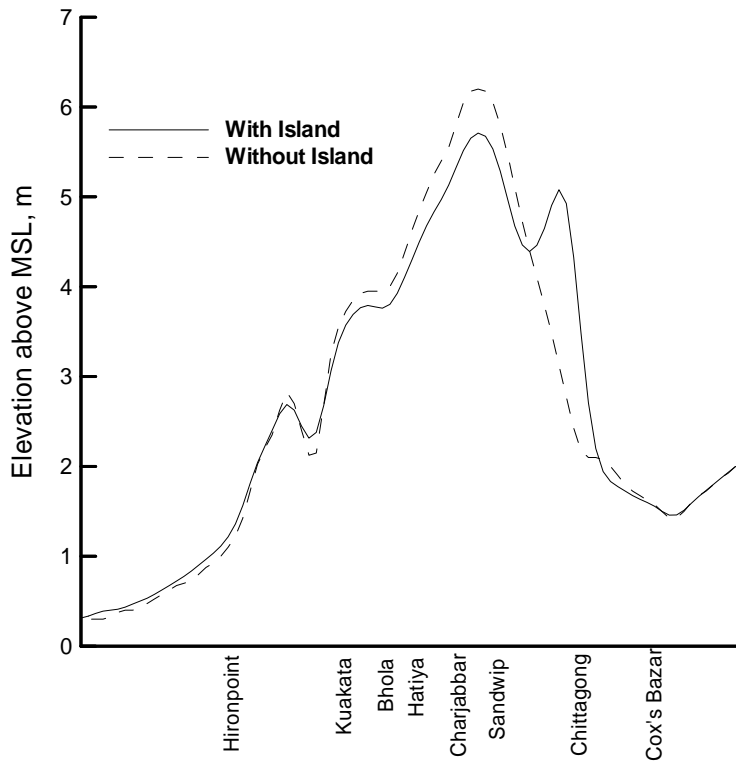


Figure 9. Maximum surge levels at different coastal locations due to 2007 storm in the presence of island and in the absence of island.

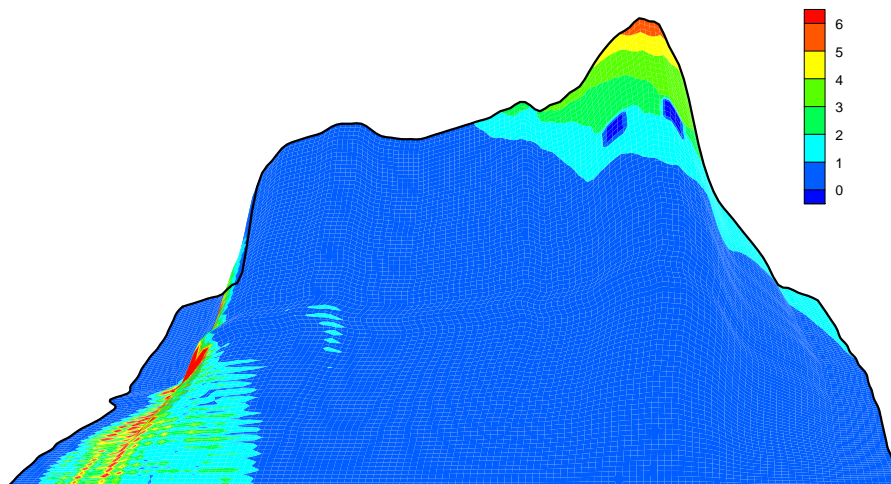


Fig 10a. Contours (in meters) of equal sea surface elevation for peak surge along the Bay of Bengal for the storm Sidr(2007).

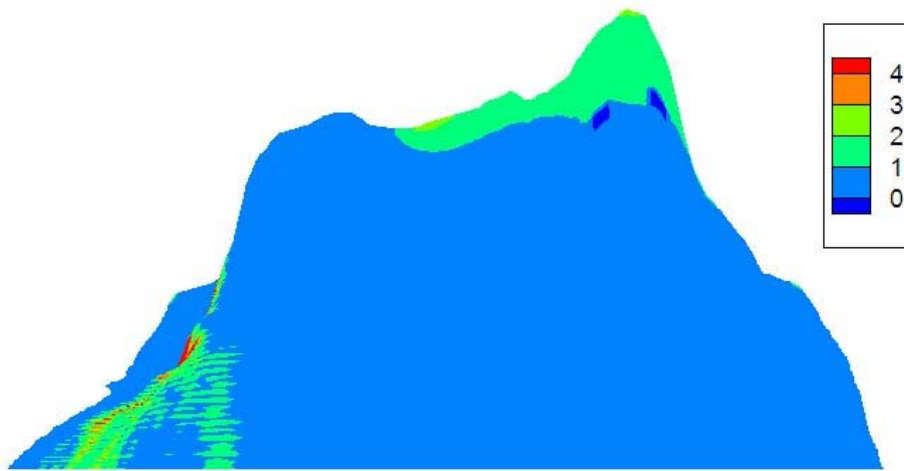


Fig 10b. Contours (in meters) of equal sea surface elevation for peak surge (without tidal consideration) along the Bay of Bengal for the storm Aila (2009).