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Approximate Algorithm for the Multi-Dimensional Knapsack Problem by Using Multiple Criteria Decision Making

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Abstract. In this paper, a new approximate method is presented to solve multi-dimensional knapsack problem by using multiple criteria decision making (MCDM). In order to, initially efficiency values for every item is calculated then items are ranked by using MCDM. Finally, items are selected in according to their rank.

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1. Introduction

The MKP is a well-known NP-Hard combinatorial optimization problem which arises in several practical problems such as capital budgeting, cargo loading, cutting stock problems, and computing processor allocation in large distributed systems. This problem is also known in the literature as the M-Dimensional Knapsack Problem, the Multi constraint Knapsack Problem, the Multi-Knapsack Problem and the Multiple Knapsack Problem. Additionally, some authors also include in their name the term zero one, e.g., the Multidimensional zero-one knapsack problem. Using alternative names for the same problem is potentially confusing but since historically, this designation Multidimensional Knapsack Problem has been the most widely used [3], we adopt this same naming.

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The practical and theoretical importance of the MKP has led to a large body of literature on both exact and approximate solution approaches. Freville [5] provides an excellent overview of the literature on the MKP and Freville and Hanafi [4] provide a survey of recently developed methods. Subsequently, Hanfi and Glover [6] offered an exploitation of nested inequalities and surrogate constraints on the MKP better than that proposed by Osorio et al. [9], but did not offer
computational results. Also, Akcay et al. [1] proposed a greedy heuristic ordering item by their value multiplied by the maximum number of copies of an item that could be accommodated with available resources.
Many real applications lead to very large scale multiple choice multidimensional knapsack problemsthat can hardly be addressed using approximate algorithms. In here, we proposed a method to solve MKP by using MCDM. This method finds an approximate solution for MKP. Proposed method uses an old idea in this field that in following explain it.

2. The Multi-Dimensional Knapsack Problem

A set of n items are packed in m knapsacks with capacities c_i. Each item j has a profit pj and weight w_{ij}associated with placing that item into knapsack i. The objective of the problem is to maximize the total profit of the selected items. The MKP is formulated as:

$$max\ \sum_{j=1}^{n} p_j x_j \tag{1}$$

$$s.t.$$

$$\sum_{j=1}^{n} w_{ij} x_j \le c_i, \quad i = 1, \dots, m \tag{2}$$

$$x_j \in \{0,1\} j = 1, \dots, n \tag{3}$$

Equation (1) calculates the total profit of selecting item j and equation (2) ensures eachknapsackConstraint is satisfied. Equation (3) is the binary selection requirement. According to (1), the goal is to choose a subset of items with maximum total profit. Selected items must, however, not exceed resource capacities; this is expressed by the knapsack constraints (2).

3. Using MCDM for MKP

The one-dimensional 0/1-knapsack problem (KP) considers items $j = 1, \dots, n$, associated profits p_j , and weights w_j . A subset of these items has to be selected and packed into a knapsack having a capacity c. The total profit of the items in the knapsack has to be maximized, while the total weight is not allowed to exceed c. Obviously, KP is the special case of MKP with $m = 1$.
The classical greedy heuristic for KP packs the items into the knapsack in decreasing order of their efficiencies$e_j = p_j/w_j$as long as the knapsack constraint is not violated.
We use this approach to solve MKP. However, because there are multi constraint for MKP, calculating items efficiency is difficult in this status. Therefore, in order to solve this problem use MCDM. Initially, efficiency corresponding to each constraint for items is compute. The efficiency of item j in constant i have a value of $E_{ij} = p_j/w_{ij}$. The below MCDM model is used to rank items[7]:

$$\begin{aligned} & E_k = max \sum_{i=1}^{m} w_i E_{ik} \\ & s.t. \quad \sum_{i=1}^{m} w_i E_{ij} \le 1, \qquad j = 1, \dots, n \\ & w_i \ge \varepsilon, \quad i = 1, \dots, m \end{aligned} \tag{4}$$

The item with bigger E_k has better rank. Now, items are selected according to their rank and constraints by following algorithm:

Algorithm

1. j=0
2. j=j+1
3. Select the j'th item according to MCDM model
4. i=1
5. While i<m+1 do
6. $x_j = 1$ in i th constraint
7. If i'th constraint satisfy
   a. i=i+1
8. else
9. Go to line 2
10. end while
11. if i=m then
12. Put j'th item in S and xj=1
13. $C_k=C_k-w_{kj}$ k=1,…,m
14. Go to line 2

4. Numerical Example

We have tested our algorithms with problems available at the OR-library [2,3] maintained by Beasley.
We solved these problems on a Pentium IV PC (2.10 GHz and 4GB of main memory) using the proposed algorithms (coded in MATLAB 7.9).

Table 1. Results on knapsack instances. For each instance, the table reports the best solutions found by Chu and Beasley as reported in [2,3](C. & B.)

n	m	Approximate method	C&B
60	30	7627	7772
28	2	140477	141278
34	4	2953	3186
100	5	22763	24381
100	5	23252	24274
100	5	22962	23551
100	10	20205	23064
100	10	19477	22801
100	10	20126	22131
500	5	114244	120134
500	5	113500	117864

The results are shown in Table 1. The first two columns indicate the sizes (n and m). The next column reports results for the proposed approximate algorithm, whereas the last columns report the known best solutions found.

5. Conclusion

The multi-dimensional knapsack problem (MKP) is an eminently difficult combinatorial optimization problem. In this paper, we present a new approximate algorithm to solve MKP based on MCDM. The proposed method is very easy to implementation. An interesting aspect of the current work is that it shows how the MCDM can be used to solve MKP. This approach may very well prove to be useful in developing fast, effective heuristics for other combinatorial optimization problems.

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