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An Electromagnetism-Like Algorithm for Fixed Charge Solid Transportation Problem

M. Sanei[a], A. Mahmoodirad[b], S. Molla-Alizadeh-Zavardehi[c,*]

[a]*Department of Mathematics, Central Tehran Branch, Islamic Azad University, Tehran, Iran.*
[b]*Department of Mathematics, Masjed Soleiman Branch, Islamic Azad University, Masjed Soleiman, Iran.*
[c]*Department of Industrial Engineering, Masjed Soleiman Branch, Islamic Azad University, Masjed Soleiman, Iran.*

Abstract. Fixed charge solid transportation problem (FCSTP) is one of the main and most important problems in transportation and network research areas. To tackle such an NP-hard problem, An Electromagnetism-like algorithm (EM) is employed. To the best of our knowledge, EM has been considered for any kind of transportation problems. Due to the significant role of parameters on the algorithm's performance, a calibration in EM is carried out with the aid of a set of experimental design. The efficiency of employed parameters is measured by the experimental design method. To evaluate the performance of the proposed EM, a computational study has been conducted and the associated results obtained by the EM are compared with simulated annealing algorithm (SA).

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*Corresponding author. Email: saber.alizadeh@gmail.com

1. Introduction

In many practical transportation problems (TP), it is realistic to suppose that the amount that can be sent on any particular route bears a fixed cost for that route. Furthermore, when a route is altogether excluded, this can be expressed by limiting its capacity to zero. In a fixed charge transportation problem (FCTP), the fixed cost is incurred for every route that is used in the solution, while the variable cost is proportional to the amount shipped. The objective is to find the combination of routes that minimizes the total variable and fixed costs while satisfying the supply and demand requirements of each origin and destination.

Fixed charge solid transportation problem (FCSTP) is an extension of the FCTP. The FCSTP deals with three type of constraints instead of two (source and destination) in the FCTP. This extra constraint is mainly due to modes of transportation (conveyance). Generally, in most real world application and problems, products are carried from origins to destinations by means of different types of conveyances (e.g., trucks, cargo flights, goods trains and ships). In other words, by considering a single type of conveyance, the FCSTP is altered to a classical FCTP.

In recent years, the solid transportation problem (STP) received much attention and many models and algorithms have been investigated. Bit et al. [2] applied fuzzy programming technique to solve the multi-objective STP. Ida et al. [5] and Gen et al. [3] considered bi-criteria and multi-criteria STP with fuzzy numbers respectively. Gen et al. [4] gave a GA for solving the bi-criteria fuzzy STP. Li et al. [12] designed a neural network approach for the multi-criteria STP. Li et al., [13] also presented an improved GA to solve a multi-objective STP with fuzzy numbers.

Jiménez and Verdegay [6] investigated a multi-objective solid transportation problem with interval data by a genetic algorithm (GA). Jiménez and Verdegay [7] investigated two types of uncertain STPs, in which the supplies, demands and conveyance capacities are intervalnumbers and fuzzy numbers. Jiménez and Verdegay [8] designed an evolutionary algorithm-based parametric approach to solve the fuzzy STP. Yang and Liu [21] presented a hybrid algorithm that is designed based on the fuzzy simulation technique and tabu search (TS) algorithm for the fuzzy FCSTP.Yang and Yuan [22] investigated a bicriteria STP under stochastic environment.

Ojha et al. [18] studied entropy based STP with general fuzzy cost and time. Ojha et al. [16] considered a stochastic discounted multi-objective STP with breakable items and applied analytical hierarchy process to solve the problem. Ojha et al. [17,19] have studied a STP with price discount and also presented single and multi-objective transportation problems using fuzzy logic. Nagarjan and Jeyaraman [15] studied the multi-objective STP with parameters as stochastic intervals. Kundu et al. [10,11] have presented two multi-objective STPs with constraints in uncertain environment. Recently, Molla-Alizadeh-Zavardehi et al. [14] developed simulated annealing (SA) and variable neighborhood search algorithms for fuzzy FCSTP.

In this paper, we consider the FCSTP. Up to now, no one has considered Electromagnetism-like algorithm (EM) for any kind of STPs. Hence, we develop and use EM for solving the STP for the first time.

The rest of the paper is organized as follows. Section 2 presents the general formulation of the FCSTP. After that, in Section 3, we propose solution representation and procedure and describe the detail of proposed EM. Thereafter, in Sections 4, to compare the solution quality of our proposed EM with the SA for solving the problem, the comprehensive computational results are presented for 140 randomly generated instances with different sizes. Finally, some concluding remarks are given in Section 5.

2. Mathematical Model and Descriptions

FCSTP can be stated as a distribution problem, in which there are m suppliers (warehouses or factories) n customers (destinations or demand points) and K conveyances (different modes of transport may be trucks, cargo flights, goods trains, ships, etc.). Each of the m suppliers can ship to any of the n customers using any of the K conveyances at a shipping cost per unit c_{ijk} (i.e., unit cost for shipping from supplier i to customer j by means of the k-th conveyance) plus a fixed cost f_{ijk}, assumed for opening this route. Each supplier$i$=1,2,…,m has a_i units of supply, each customer j=1,2,…,n has a demand of b_j units and each conveyance k=1,2,…,K has a capacity of e_k units. The objective is to determine, in which routes are to be opened and the size of the shipment on those routes using conveyances in such a way that the total cost of the met demand is minimized while satisfying the supply and shipment capacity constraints. The standard FCSTP formulation is shown below.

$$Min\ \ Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}(c_{ijk}\ x_{ijk} + f_{ijk}\ y_{ijk}) \tag{1}$$

s.t.

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le a_i \qquad i = 1,...,m \tag{2}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge b_j \qquad j = 1,...,n \tag{3}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_k \qquad k = 1,...,K \tag{4}$$

$$x_{ijk} \ge 0 \qquad i = 1,...,m; j = 1,...,n;\ k = 1,...,K \tag{5}$$

$$\begin{matrix} y_{ijk} = 0 & \text{if} & x_{ijk} = 0 \\ y_{ijk} = 1 & \text{if} & x_{ijk} > 0 \end{matrix} \qquad i = 1,..., m; j = 1,..., n;\ k = 1,..., K \tag{6}$$

where $x_{ijk}$is the unknown quantity to be transported on the route *(i,j)* that from plant i to consumer j by means of conveyance k, $c_{ijk}$is the shipping cost per unit from plant i to consumer j by means of conveyance k. $a_i$is the number of units available at plant i, b_j is the number of units demanded at costumer j and e_k is the unit of this product called conveyances that can be carried by K different mode of transportation. The transportation cost for shipping per unit from plant i to consumer j using conveyance k is $c_{ijk} \times x_{ijk}$. f_{ijk} is the fixed cost associated respectively. In this paper, we assume a balanced transportation problem, because the unbalanced transportation problem can be converted to a balanced transportation problem by introducing a dummy plant, dummy consumer or dummy conveyance.

3. Proposed Electromagnetism-Like Algorithm

The EM was first introduced to simulate the electromagnetism theory of physics by Birbil and Fang [1] as a new stochastic population-based heuristic optimization tool to solve the problems with lower and upper bound in the form of:

$$Min \qquad f(x) \qquad (7)$$
$$s.t. x \in [L,U] \qquad (8)$$

Where $[L,U] = \{x \in R^n \mid L_k \leq x_k \leq U_k, k = 1,...,n\}$ and $x_1,...,x_n$ represent the decision variables. U_k and L_k represent, upper and lower bounds on the kth variable (k = 1,…, n), respectively, and f (x) is objective function value.

EM uses the attraction–repulsion mechanism of the electromagnetism theory to put the sample solutions toward to the optimal solution. By the Coulomb's law, the amount of force is proportional to the product of the particle's charge and inversely proportional to the distance between them. The positions of them are calculated iteratively according to the resultant force exerted by a population of other charged particles. The idea behind the algorithm is that inferior particles prevent a move in their direction by repelling other superior particles and that better particles facilitate moves in their direction. So, the force causes a global movement of all solutions towards the solutions with higher quality.

The general structure of the EM algorithm is described in Algorithm 1. The EM algorithm has four main stages. The first step is determination of the initial solutions. In the first phase procedure, population size (popsize) of solutions are randomly generated from the feasible region. The attribute of solutions is assumed to be uniformly distributed among the corresponding upper bound and lower bound.

After initialization, the second step is to conduct a local search to improve the solution's quality. The third step is to calculate the total force exerted on each particle according to their charges. The final step includes moving along the direction of the force. After computing the total force of one particle, this particle moves the random step length in the lane of the force to cause the particles to move into any new feasible region along this lane which is uniformly distributed between 0 and 1. The changeable value of every dimension is limited to the corresponding lower upper and bounds. The specific formulas for the FBPM used to calculate charges, forces and the movement action of each solution will be described in Sections 3.3 and 3.4.

ALGORITHM 1.**EM** (popsize, MAXITER, LSITER)
l: lnitialize (popsize)
2: iteration ← l
3: **while** termination criterion are not satisfied **do**
4: Local search (LS1TER)
5: Calculate Forces
6: Move
7: iteration ← iteration + l
8: **end while**

3.1. Encoding Scheme and Initialization

The random key method is used for solving the problem. The length of solutions vector is equal to the summation of numbers of suppliers, customers and conveyances ($m + n + K$). To generate a solution by this method, random real numbers between zero and one are generated for each position. By ascending sorting of the value corresponding to each position of subsection, the related discrete solution vector is obtained. It is easy to see that the optimal solution should be obtained under the condition that the demand of each destination, supply abilities of each plant and conveyance capacities are just satisfied. So the following method is employed to decode the solution.

Procedure: initialization
Repeat the following process until no digits are left in solution vector:
select leftmost digit i from subsection 1 of set π;
select leftmost digit j from subsection 2 of set π;
select leftmost digit k from subsection 3 of set π;
Assign available amount of units to $x_{ijk} = \min\{a_i, b_j, e_k\}$;
Update availability $a_i = a_i - x_{ijk}$, $b_j = b_j - x_{ijk}$ and $e_k = e_k - x_{ijk}$).
If $a_i = 0$, remove the digit i to the solution.
If $b_j = 0$, remove the digit j to the solution.
If $e_k = 0$, remove the digit k to the solution.

3.2. Local Search

The procedure that selects each near random solution (Algorithm 2, lines 4–12) then finds it's related their objective value. This new selected solution will replace the current solution when its quality is better than the current solution (Algorithm 2, lines 13–16). Finally the current best point is updated (Algorithm 2, line 21).

ALGORITHM2. Local (LSITER)

```
1: counter ←1
2: for i = 1 to popsize do
3:          for k = 1 to n do
4:                  λ1 ← U (0, 1)
5:                  while counter <LSITER do
6:                                 Y ← Xi
7:                                 λ2 ← U (0, 1)
8:                                 if λ1> 0.5 then
9:                                      Yk ← Yk + λ2
10:                              else
11: Yk ← Yk− λ2
12:                              end if
13:                              if f (Y) < f (Xi) then
14:                                   Xi ← Y
15:                                   counter ← LSITER − 1
16:                              end if
17:                              counter ←counter + 1
18:                 end while
19:         end for
20: end for
21: Xbest ← argmin{f (Xi), ∀i}
```

3.3. Total Forces Computation

The charge q^i and the total force vector exerted on X^i computed by the superposition principle is

$$F_j^i = \sum_{\substack{k=1 \\ k \neq i}}^{popsize} \left\{ \begin{array}{ll} \left(x_j^k - x_j^i\right) \dfrac{q^i q^k}{\left\| x^k - x^i \right\|^2} & if \quad f\left(X^k\right) < f\left(X^i\right) \\ \left(x_j^i - x_j^k\right) \dfrac{q^i q^k}{\left\| x^k - x^i \right\|^2} & if \quad f\left(X^k\right) \geq f\left(X^i\right) \end{array} \right\}, \quad i = 1, \ldots, popsize, \ j \in J \qquad (9)$$

where $f(X^k) \geq f(X^i)$ represents attraction and $f(X^k) < f(X^i)$ represents repulsion. After comparing the objective values, the direction of the move between the particles is determined. Therefore, X^{best} plays the role of attraction, i.e., it attracts all particles in the population.

$$q^i = \exp\left(-n \frac{f(X^i) - f(X^{best})}{\sum_{k=1}^{popsize} \left(f(X^i) - f(X^{best}) \right)} \right), \quad i = 1, \ldots, popsize, \ j \in J \tag{10}$$

$$\left\| x^k - x^i \right\| = \left(\sum_{j \in J} \left(x_j^k - x_j^i \right)^2 \right)^{1/2} \tag{11}$$

And X^{best} is the best solution in the current population.

3.4. Movement Procedure

After evaluating the total force vector F^i, particle X^i moves in the direction of the total force by a random step length, i.e.,

$$x_j^i = x_j^i + \lambda \frac{F_j^i}{\left\| F^i \right\|} (RNG_j) \quad i = 1, \ldots, popsize, \ j \in J \tag{12}$$

Where RNG_j denotes the amount of feasible movement toward the zero or one and the random step length λ = random (0, 1).

Since RKs are real numbers between zero and one, the adaptation of Eq. (12) for the RKs gives the following formula:

$$x_j^i = \begin{cases} x_j^i + \lambda \frac{F_j^i}{\left\| F^i \right\|} \left(1 - x_j^i \right) & if \quad F_j^i > 0 \\ x_j^i + \lambda \frac{F_j^i}{\left\| F^i \right\|} \left(x_j^i \right) & if \quad F_j^i \leq 0 \end{cases} \quad i = 1, \ldots, popsize, \ j \in J \tag{13}$$

where

$$\left\| F^i \right\| = \left(\sum_{j \in J} F_j^{i^2} \right)^{1/2}. \tag{14}$$

Note that the current best particle does not move because of having the better objective value and attracting all other particles.

4. Experimental Design

4.1. Instances

In this subsection Instances generation are conducted to set the parameters and evaluate the performances of proposed algorithms. The data required for a problem consists of the number of suppliers, customers and conveyances, total demand, and range of variable costs and fixed costs [13]. For running the algorithms, 28 problem sets are generated at random, in which seven problem sizes are implemented for the experimental study. The problem size is determined by the number of suppliers, customers and conveyances. The problem details are shown in Table 1.

Table 1. Test problems characteristics.

			Range of variable costs		Range of first and second fixed costs	
Problem size	Total Demand	Problem type	Lower limit	Upper limit	Lower limit	Upper limit
10×10×4	10,000	A	3	8	50	200
10×20×4	15,000	B	3	8	100	400
15×15×6	15,000	C	3	8	200	800
10×30×6	15,000	D	3	8	400	1,600
50×50×8	50,000					
30×100×8	30,000					
50×200×10	50,000					

4.2. Experimental Results

A computational study was conducted to evaluate the efficiency and effectiveness of the proposed algorithm, which was coded in MATLAB and run on a PC with 2.8 GHz Intel Core 2 Duo and 4 GB of RAM memory. For this purpose, we present and compare the results of EM with the SA algorithm as an effective algorithm in the literature.

We use searching time as stopping criterion to be equal for both algorithms which is equal to $1.4 \times (n + m + K)$ milliseconds. Therefore, CPU time is affected by all the problem characteristic n, m and K. The more the number of suppliers, customers and conveyances, the more the rise of CPU time increases. For further comparison, the convergence is investigated. The best results and their convergence are showed in Fig. 1. The superiority of EM on SA is clear. From this figure, it is concluded that EM has a better convergence than SA on this problems.

We generated five test problems for each twenty eight problem type, summing to $28 \times 5 = 140$ instances. Because the scale of objective functions in each instance is different, they could not be used directly. To solve this problem, the relative percentage deviation (RPD) is used for each instance. The RPD is obtained by:

$$RPD = \frac{Alg_{sol} - Min_{sol}}{Min_{sol}} \times 100$$

where$Alg_{sol}$and Min_{sol} are the obtained objective value and minimum objective value found from both proposed algorithms for each instance, respectively.The problems have been run ten times and the averages of RPDs for each algorithm and each problem size are showed in Fig. 2. As it is obvious, EM exhibits robust performance, meanwhile the problems size increases. It also shows remarkable performance improvements of EM in all size problems versus SA.

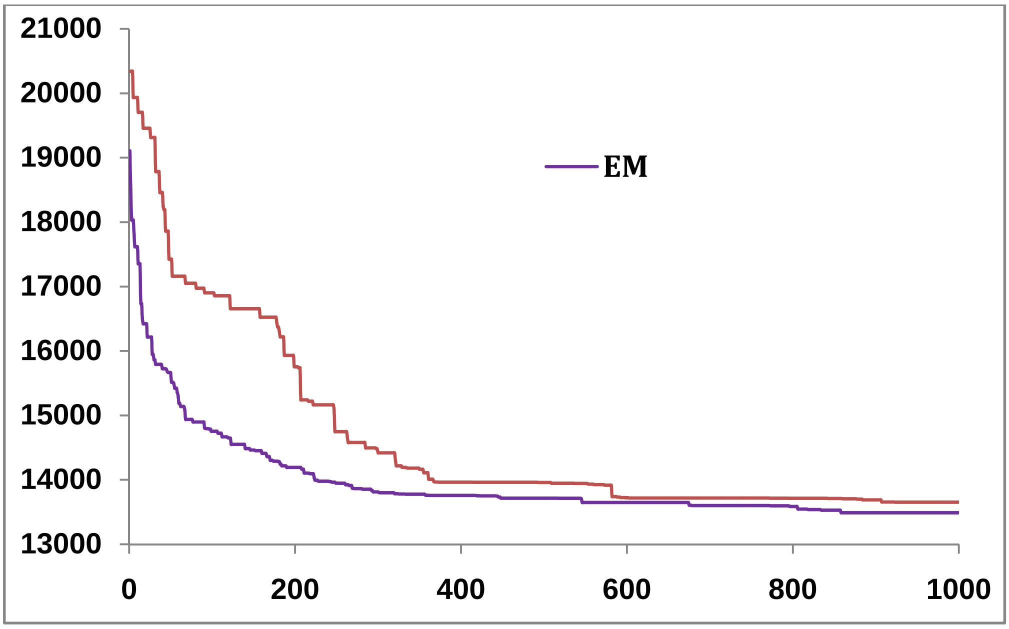

Figure 1. Convergence of EM and SA Algorithms.

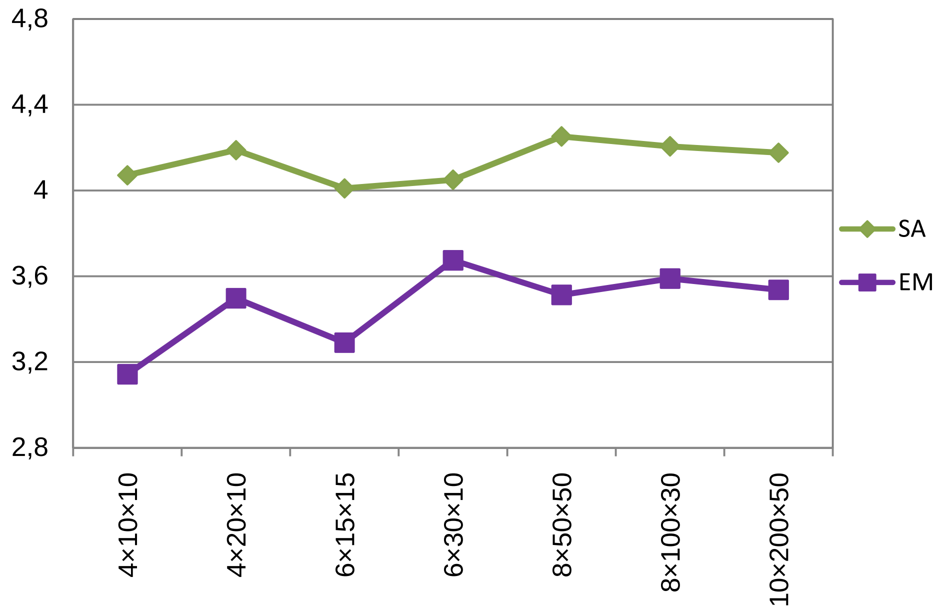

Figure 2. Means plot for the interaction between EM and SA in each problem size

5. Conclusion and Future Works

A fixed charge solid transportation problem (FCSTP) is one of the NP-hard problems that are difficult to solve by traditional methods. In this paper, we have proposed an efficient Electromagnetism-like algorithm (EM) with new solution representation and procedure to solve the problem. To comprehensively compare the results obtained by the EM and SA in terms of the solution quality, we have established an experiment with 28 different problem type and 140 randomly generated instances with different sizes. The computational results have shown the superiority of the EM in comparison with SA. There still exist rich opportunities for researchers to further the study in this area. The future work is to extend our approach to the case of inventory cost [9] or fuzzy numbers [20] for solving real life distribution problems. It seems that EM has the capability of achieving better results for large-sized problems if it hybridizes with other suitable single solution metaheuristics like variable neighborhood search or local search methods; therefore it can be considered in future work. Also, it can be interesting to investigate and develop new algorithms based on other metaheuristics and compare them with our algorithms.

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