

## Mathematical Modelling for Dice Finder Game Problem

Sanjay Jain<sup>a</sup> and Chander Shakher<sup>b</sup>

<sup>a</sup>Department of Mathematical Sciences, Government College, Ajmer  
Affiliated to M. D. S. University Ajmer, Ajmer-305001, India.

<sup>b</sup>Research Scholar, Mahatma Gandhi University, Meghalaya.

---

**Abstract.** Play is often episodic and mission-centric, with a series of challenges culminating in a final puzzle or enemy that must be overcome. Multiple missions played with the same characters may be related to each other in a plot arc of escalating challenges. The exact tone, structure, pace and end (if any) vary from game to game depending on the needs and preferences of the players. "THE CHANCE ELEMENT in thousands of indoor games is introduced by a variety of simple random-number generators. The most popular of such devices, ever since the time of ancient Egypt, have been cubical Dice. Cubical, Because of their symmetry, any of the five regular solids can be and have been used as gaming dice, but the cube has certain obvious advantages over the other four solids. It is the easiest to make, its six sides accommodate a set of numbers neither too large nor too small, and it rolls easily enough but not too easily. In our proposed Dice Finder game problem, user can see the different dice number's on screen from 1 to 6. Using the property of Dice that "Opposite sides of a dice add up to seven", user has to find that correct number of dice out all of that, for which one condition is true.

---

Received: 25 May 2013; Revised: 3 September 2013; Accepted: 7 October 2013.

**Keywords:** Dice, Die, Fair dice, Tabletop role-playing game, Dice finder.

### Index to information contained in this paper

1. Introduction
2. Terminology Used
3. Game play
4. Mathematical Approach
5. Method & Algorithm
6. Dice Probabilities
7. Diagrammatic representation of Dice Finder Game
8. Conclusion

### 1. Introduction

A dice, "something which is given or played", is a small throw able object with multiple resting positions, used for generating random numbers. This makes dice suitable as gambling devices for games like craps, or for use in non-gambling tabletop games. A

---

\*Corresponding author. Email: [drjainsanjay@gmail.com](mailto:drjainsanjay@gmail.com)

traditional dice is a rounded cube, with each of its six faces showing a different number

of dots from 1 to 6. When thrown or rolled, the dice comes to rest showing on its upper surface a random integer from one to six, each value being equally likely. A variety of similar devices are also described as dice; such specialized dice may have polyhedral or irregular shapes and may have faces marked with symbols instead of numbers. They may be used to produce results other than one through six. Loaded and crooked dice are designed to favor some results over others for purposes of cheating or amusement, as in [5 & 21]. Traditionally, opposite sides of a dice add up to seven, implying that the 1, 2 and 3 faces share a vertex; as in [8], these faces may be placed clockwise or counter clockwise about this vertex. If the 1, 2 and 3 faces run counter clockwise, the dice is called right-handed and vice versa. Western dice are normally right-handed and Chinese dice are normally left-handed, as in [7].

Dice are usually some sort of regular polyhedra. Best known are the Pythagorean solids, the classical d4, d6, d8, d12, and d20, but other shapes are also used, primarily the d10. In addition to these, a d30 and a d100 are available in well-stocked game stores. A consideration of symmetry tells us that all of these except the d100 are "proper" dice since all of their faces are equivalent. The d100 is less certain, since its surfaces aren't clearly defined. The questions I'm going to answer is, can this d100 be a proper dice (with clearly equal chances of landing on all faces)? Are there more proper dice than those we know? And, can you make dice with curved surfaces that can't be made with flat surfaces?

The requirements for a die with a certifiably equal chance of landing on each face, are that all the faces must be identical (though mirror symmetry is allowed), and must be placed identically in relation to each other. In other words, the faces on the die must be indistinguishable barring mirror symmetry. The die must be also be convex (it can't curve in on itself), or it would be unable to land on some faces.

## **2. Terminology Used**

### **2.1 Fair Dice**

Dice are usually cubes of a homogeneous material. Symmetry suggests that a homogeneous cube has the same chance of landing on each of its six faces after a vigorous roll, so it is said to be fair. Similarly the four other regular solids-the tetrahedron, octahedron, dodecahedron and icosahedron-are fair. A convex polyhedron is fair by symmetry if and only if it is symmetric with respect to all its faces. This means that any face can be transformed into any other face by a rotation, a reflection, or a combined rotation and reflection, which takes the polyhedron into itself. The collection of all these transformations of a given polyhedron is called its symmetry group. The fact that some

transformation in the group takes any given face into any other given face is expressed by saying that the group acts transitively on the faces. Thus we can say that a convex polyhedron is fair by symmetry if and only if its symmetry group acts transitively on its faces, as in [17].

Here we shall determine all such polyhedra and in the final section we shall show that there are other polyhedra which are fair, but not fair by symmetry.

## 2.2 Polar reciprocals

All the symmetry transformations of a convex polyhedron leave invariant its center of gravity, which is a point inside the polyhedron. We form the dual or polar reciprocal polyhedron of the original polyhedron with respect to its center. This is done by passing a plane through each vertex of the original polyhedron, perpendicular to the line from the center to the vertex. These planes form the faces of the polar reciprocal polyhedron, and each of its vertices is called a pole of one face of the original polyhedron. Because the symmetry group acts transitively on the faces of the original polyhedron, it also acts transitively on their poles, which are the vertices of the polar reciprocal polyhedron. Therefore the polar reciprocal polyhedron is symmetric with respect to its vertices.

The symmetry groups of all polyhedra symmetric with respect to their vertices have been determined, as in [4]. Furthermore for each such group there is one polyhedron which has regular polygons for faces. These particular polyhedra are the well-known semi-regular solids. They comprise thirteen individuals, the Archimedean solids, and the two infinite classes of prisms and anti-prisms, which were recognized as semi-regular by Kepler. The regular or Platonic solids are also semi-regular.

Now we use the fact that the polar reciprocal of the polar reciprocal of a polyhedron is similar to the original polyhedron. Therefore we have obtained the following result: The polyhedra which are fair by symmetry are duals of the polyhedra symmetric with respect to their vertices. Each symmetry group of a fair polyhedron is represented by a regular solid or the dual of a semi regular solid. Thus in addition to the five regular solids there are thirteen individual polyhedra and two infinite classes among the fair polyhedra.

Regular and semi-regular solids are listed in [4], together with some of their properties. From the list, we see that every semi-regular solid has an even number of vertices, so its polar reciprocal has an even number of faces. Therefore every polyhedron fair by symmetry has an even number of faces. Drawings of the polar reciprocals of the semi-regular solids as in [21], and photographs of models of them are shown as in [1].

Each of the semi-regular solids belongs to a class of polyhedra with the same symmetry group, all of which are symmetric with respect to their vertices. The dual of any one of them is fair by symmetry, but in general it will be less symmetric than the dual of the semi-regular solid. For example, suppose that each equilateral triangular face of a semi-regular antiprism is replaced by a given isosceles triangle which is not equilateral. The resulting solid is symmetric with respect to its vertices and has the same symmetry group as the semi-regular antiprism. Its dual is a fair dice with quadrilateral faces which are not

symmetric about either of their diagonals. On the other hand, suppose that the square faces of a semi-regular prism are replaced by non-square rectangles. The new polyhedron is symmetric with respect to its vertices and has the same symmetry group as the semi-regular prism. Its dual is again a dipyramid which is fair by symmetry. It differs from the dual of the semi-regular prism only in the ratio of the lengths of the sides of a face. Grünbaum and Shephard [2], give a complete classification of the convex polyhedra with symmetries acting transitively on their faces, which they call "isohedra." They are classified by combinatorial isomorphism type and by symmetry group. There are some isohedra for which the positive rotations alone (i.e. those which preserve orientation, so they do not include reflections) do not act transitively on the faces. Earlier Grünbaum [3] showed that every isohedron has an even number of faces.

### 2.3 Other fair polyhedra

There are other fair polyhedra which are not symmetric. To show this we consider, for example, the dual of the  $n$ -prism, which is a dipyramid with  $2n$  identical triangular faces. We cut off its two tips with two planes parallel to the base and equidistant from it. When the cuts are near the tips, the solid has a very small probability of landing on either of the two tiny new faces. However when the cuts are near the base, it has a very high probability of landing on one of them. Therefore by continuity there must be cuts for which the two new faces and the  $2n$  old faces have equal probabilities. The locations of those cuts, which depend upon the mechanical properties of the die and the table, could be found by experiment or by a difficult mechanical analysis along the lines of our previous study of coin tossing [19]. Similar constructions can be carried out starting with other dice which are fair by symmetry, or obtained by this construction. These dice are fair by continuity. Example: let us consider an infinite prism with a regular  $n$ -gon as its cross section. Let us cut it with two planes at distance  $L$  apart perpendicular to its generators, to produce a polyhedron with  $n + 2$  faces. For  $L$  large this solid has very low probability of landing on either of its two ends, whereas, for very small  $L$  it has a high probability of landing on one of them. Therefore by continuity there is some value of  $L$  for which it has the same probability of landing on any one of its  $n + 2$  faces. When  $n$  is odd this yields a fair die with an odd number of faces.

The problem of characterizing all fair dice, not just those which are fair by symmetry or by continuity, is still unsolved.

### 2.4. Narrative or Tabletop RPGs

Narrative or Tabletop Role-playing Games are a large class of commercially-available games. These are usually available only at specialized hobby or game stores, although a few (such as Dungeons & Dragons) can be found in regular bookstores. "Narrative" means that game actions are taken primarily through verbal declaration (i.e. "my character climbs the wall"). These are also known as "tabletop" RPGs (to distinguish them from live-action roleplaying) or "paper-and-pencil" RPGs (to distinguish them from computer games).

Narrative role-playing games are played sitting around in a comfortable setting (often around a table but not necessarily), and what happens is defined by verbal description. i.e. A player simply declares "I am walking to the window", and it is understood that her character is doing just that. Diagrams and notes may be used as aids, but narration is the primary medium. These are often referred to as "tabletop" RPGs (to distinguish them from "live action" RPGs where the players move around) or "paper-and-pencil" RPGs (to distinguish them from computer games), as in [13].

#### 2.4.1 Application in role-playing games

The fantasy role-playing game Dungeons & Dragons (D&D) is largely credited with popularizing dice in such games. Some games use only one type, like Exalted which uses only ten-sided dice. Others use numerous types for different game purposes, such as D&D, which makes use of all common polyhedral dice.

Dice are used to determine the outcome of events; such usage is called a *check*. Games typically determine results either as a total on one or more dice above or below a fixed number, or a certain number of rolls above a certain number on one or more dice. Due to circumstances or character skill, the initial roll may have a number added to or subtracted from the final result, or have the player roll extra or fewer dice. To keep track of rolls easily, dice notation is frequently used, as in [6].

#### 2.5 Dice notation

(Also known as **dice algebra**, **common dice notation**, **RPG dice notation**, and several other titles) is a system to represent different combinations of dice in role-playing-games using simple algebra-like notation such as  $2d6+12$ , as in [6].

A common special case is percentile rolls, referred to as  $1d100$  or  $1d\%$ . Since actual hundred-sided dice are large, almost spherical, and difficult to read, percentile rolls are instead handled by rolling two ten-sided dice together, using one as the "tens" and the other as the "units". A roll of ten or zero on either die is taken as a zero, unless both are zeros or tens, in which case this is 100. Some sets of percentile dice explicitly mark one die in tens and the other in units to avoid ambiguity.

#### 2.6 Tabletop role-playing game

A **tabletop role-playing game**, **pen-and-paper role-playing game**, or **table-talk role-playing game** is a form of role-playing game (RPG) in which the participants describe their character's actions through speech. Participants determine the actions of their characters based on their characterization, as in [12] and the actions succeed or fail according to a formal system of rules and guidelines. Within the rules, players have the freedom to improvise; their choices shape the direction and outcome of the game, as in [13].

We are stuck with the fact that people use the phrase 'role-playing' in at least three different senses; as in [15]

To refer, to the playing of roles generally; in life, in theatre, in the consulting room.

To refer, to a wide range of board games, computer games, PBM games and live action games in which players control the actions of fictional characters.

To refer, specifically, to a type of interactive narrative of which Amber, Shadow run and Shatter-zone are examples.

### 3. Game play

Most games follow the pattern established by the first published role-playing game, Dungeons & Dragons. Participants usually conduct the game as a small social gathering. One participant, called the Dungeon Master (DM) in *Dungeons and Dragons*, more commonly called the Game Master or GM, purchases or prepares a set of rules and a fictional setting in which players can act out the roles of their characters. This setting includes challenges for the player characters to overcome through play, such as traps to be avoided or adversaries to be fought. The full details of the setting are kept secret, but some broad details of the game world are usually given to the players. Games can be played in one session of a few hours, or across many sessions depending on the depth and complexity of the setting.

The players each create characters whose roles they will play in the game. As well as fleshing out the character's personal history and background, they assign numerical statistics to the character; these will be used later to determine the outcome of events in the game. Together, these notes tell the player about their character and his or her place in the game world, as in [13].

The GM then begins the game by introducing and describing the setting and the characters. The players describe their characters' actions, and the GM responds by describing the outcome of those actions. Usually, these outcomes are determined by the setting and the GM's common sense; most actions are straightforward and immediately successful, as in [13]. The outcomes of some actions are determined by the rules of the game. In some game systems, characters can increase their attribute scores during the course of the game (or over multiple games) as the result of experience gained. There are alternate games systems which are dice-less, or use alternate forms of randomization, such as the non-numerical dice of Fudge or a Jenga tower, as in [16].

Games are of indefinite length, from a single brief session to a series of repeated sessions that may continue for years with an evolving cast of players and characters. Multiple missions played with the same characters may be related to each other in a plot arc of escalating challenges. The exact tone, structure, pace and end vary from game to game depending on the needs and preferences of the players.

#### 3.1 Game systems

The set of rules of a role-playing game is known as its *game system*; the rules themselves are known as game mechanics. Although there are game systems which are shared by many games, for example the d20 system, many games have their own, custom rules system.

Many role-playing games require the participation of a game master (GM), who creates a setting for the game session, portrays most of its inhabitants, known as non-player

characters (NPCs) and acts as the moderator and rules arbitrator for the players. The rest of the participants create and play inhabitants of the game setting, known as player characters (PCs). The player characters collectively are known as a "party".

During a typical game session, the GM will introduce a goal for the players to achieve through the actions of their characters. Frequently, this involves interacting with non-player characters, other denizens of the game world, which are played by the GM. Many game sessions contain moments of puzzle solving, negotiation, chases, and combat. The goal may be made clear to the players at the outset, or may become clear to them during the course of a game.

Some games, such as *Polaris* and *Primetime Adventures*, have distributed the authority of the GM to different players and to different degrees. This technique is often used to ensure that all players are involved in producing a situation that is interesting and that conflicts of interest suffered by the GM are avoided on a systemic level.

Game rules determine the success or failure of a character's actions. Many game systems use weighted statistics and dice rolls or other random elements. In most systems, the GM uses the rules to determine a target number though often the targets are determined in a more principled fashion. The player rolls dice, trying to get a result either more than or less than the target number, depending on the game system. Not all games determine successes randomly, however; an early and popular game without random elements is *Amber Diceless Roleplaying Game* by Erick Wujcik.

Most systems are tied to the setting of the game they feature in. However, some universal role-playing game systems can be adapted to any genre. The first game in 1981; as in [3] to feature such a system, *Champions*, is accompanied by a number of sourcebooks which allow games to be created in different genres. The *d20 system*, based on the older role-playing game *Dungeons and Dragons*, is used in many modern games such as *Spycraft* and the *Star Wars Roleplaying Game*.

In practice, even universal systems are often biased toward a specific style or genre and adaptable to others. For example, although the d20 system has sourcebooks for modern and futuristic settings, most published d20 system material stays within *Dungeons & Dragons'* combat-focused fantasy milieu.

### **3.2 Business models**

Role-playing games are produced under a variety of business models, which succeed or fail based on those models' objectives. The smallest viable businesses are one person companies that produce games using print on demand and e-book technologies. Most of these companies provide a secondary income for their owner-operators. Many of these businesses employ freelancers, but some do not; their owners complete every aspect of the product. Larger companies may have small office staff that manages publishing, brand development and freelance work. Guided by a developer/manager, freelancers produce most of a game line's content according to a central plan. Finally, a few companies (such as *Wizards of the Coast* and *Mongoose Publishing*) maintain an in-house writing and design staff.

The standard business model for successful RPGs relies on multiple sales avenues:

- The so-called three-tier distribution model, under which the company sells products to distributors who in turn sell the products to retailers who sell to customers. This is traditionally divided into the hobby trade (used by the majority of print publishers) and the book trade (viable for a smaller number of companies able to absorb returns and provide sufficiently large print runs). The industry consensus is that hobby retail sales have greatly declined, with the balance of hobby games sales moving from RPGs to miniatures games and collectible card games.
- Direct sales via the internet, through an online retailer or through the company's own electric storefront.
- Electronic sales and distribution, either without any physical product at all (e-books) or through a POD service. Once limited to small companies, this sales venue is now employed by publishers of all sizes.
- Attendance at conventions and events; this is particularly common among live-action games.

Typically, RPG publishers have a very long life cycle once they manage to generate an initial successful game. TSR, the initial publisher of Dungeons & Dragons was an independent entity until 1997 when it was acquired by Wizards of the Coast, who was subsequently acquired by Hasbro in 1999. Many of TSR's contemporaries remain in business as independent publishers. The core design group of a publisher is often kept as a team within the new company for the purposes of continuity and productivity, though layoffs are common after such mergers and acquisitions. For example, Wizards of the Coast experienced multiple layoffs in the wake of acquiring Last Unicorn Games and after its own acquisition by Hasbro.

#### 4. Mathematical Approach

We now turn to our mathematical approach. This isn't too complex, but a bit time-consuming, so you may want to jump directly to the results. As we all know, Euler's equation, which is good for any convex polyhedron of three or more faces (curved or otherwise), states that

$$V + N = E + 2$$

Where, V is the number of vertices in the polyhedron,

N the number of faces,

and E the number of edges.

Since the faces on a proper die are identical, they must all have the same number of sides (and corners), a number we'll call M. We have  $E = M*N/2$ , which tells us that N only can be odd if M is even (or E won't be a whole number), as in [20].

Using this substitution, Euler's equation becomes:

$$V = N (M / 2 - 1) + 2$$

As per dice property: **Opposite sides of a dice add up to seven**, user has to find that correct number of dice out all of that, for which one condition is true, i.e

a) The total of left and right number's,

b) The total of upper and lower number's, of that correct number is seven.

If user tap on that correct number in once, user will get a message “AapJeetGye !” or “You Win !”. There is also timer for the player to complete the game in specific time interval, if user/player can't complete the game in given time interval, he will loose the game.

Here is an example:



In above example middle one '6' is our correct number. The total of 3+4 & 2+5 is 7.

## 5. Method & Algorithm

Gamers use all sorts of dice. Dice are usually some sort of regular polyhedra. Best known are the Pythagorean solids, the classical d4, d6, d8, d12, and d20, but other shapes are also used, primarily the d10. In addition to these, a d30 and a d100 are available in well-stocked game stores. A consideration of symmetry tells us that all of these except the d100 are "proper" dice since all of their faces are equivalent. The d100 is less certain, since its surfaces aren't clearly defined. The questions I'm going to answer is, can this d100 be a proper dice (with clearly equal chances of landing on all faces)? Are there more proper dice than those we know? And, can you make dice with curved surfaces that can't be made with flat surfaces?

The requirements for a die with a certifiably equal chance of landing on each face, are that all the faces must be identical (though mirror symmetry is allowed), and must be placed identically in relation to each other. In other words, the faces on the die must be indistinguishable barring mirror symmetry. The die must be also be convex (it can't curve in on itself), or it would be unable to land on some faces.

We now turn to the math. This isn't too complex, but a bit time-consuming, so you may want to jump directly to the results.

Euler's equation, which is good for any convex polyhedron of three or more faces (curved or otherwise), states that  $V + N = E + 2$ , where  $V$  is the number of vertices in the polyhedron,  $N$  the number of faces, and  $E$  the number of edges. Since the faces on a proper die are identical, they must all have the same number of sides (and corners), a number we'll call  $M$ . We have  $E = M*N/2$ , which tells us that  $N$  only can be odd if  $M$  is even (or  $E$  won't be a whole number), as in [10].

**5.1** Using this substitution, Euler's equation becomes:  $V = N(M/2 - 1) + 2$

This equation does not cover the situations where  $M=0$  or  $M=1$ . These, however, only have one solution each, namely:

**5.1.1**  $N=1, M=0, V=0$  (a sphere)

**5.1.2**  $N=2, M=1, V=0$  (a lens)

In the case  $M=2$ , 8.1 simplifies to  $V = 2$ . This has solutions for all  $N$  greater than 2, but requires curved surfaces. The shape of these "dice" is prisms that taper in both ends, with  $N$ -sided cross sections:

**5.1.3**  $N \geq 3, M=2, V=2$  (edged cigar shapes)

As will be shown later, this is the only way to make dice with an odd number of faces (notcounting the sphere).

We turn to the cases  $M > 2$ . Since a vertex must contain at least three face corners, we have that  $3V \leq N*M$ , which with 8.1 tells us that  $M \leq 6 - 12/N$ , which means that  $M < 6$ . The faces must thus be triangles, quadrangles, or pentagons.

While we require that all the faces on the dice are equivalent, the same is not true for the vertices. There can theoretically be as many types of vertices as there are types of corners in the faces. Since a given type of corner must always be part of the same type of vertex (or the faces wouldn't be equivalent), there can't be more types of vertices than there are corners, but there can be less (several types of corners may meet in one type of vertex).

Let's label the different types of vertices with a number  $i, i=1,2,\dots$  up to the number of different vertices. Let's call the number of corners that meet in a vertex the *rank* of the vertex, and label that rank  $R_i$ . Similarly, we label the number a vertex type occurs  $V_i$ , and the number of different corners a vertex is made out of  $M_i$ .

We get:

**5.2**  $V = \text{SUM}(V_i) (i=1,2,\dots)$

**5.3**  $M = \text{SUM}(M_i) (i=1,2,\dots)$

**5.4**  $V_i = N*M_i/R_i$

The two first are obvious: The total number of vertices is the sum of the numbers of the individual vertices, and the total number of corners on a face is the sum of the corners that are part of different vertices. 5.4 must be true since all the corners in vertices must be provided by the faces, and vice versa.

Combining 5.2, 5.3, and 5.4 with 5.1 gives us:

**5.5**  $\text{SUM}(N*M_i/R_i) = N(M/2 - 1) + 2$  Which reduces to:

**5.6**  $N = 2/(\text{SUM}(M_i/R_i) - M/2 + 1)$

In the cases where there's only one type of vertex we have  $M_i=M$ , and 5.6 becomes

$N = 2/(M/R - M/2 + 1)$ , which has these solutions:

**5.6.1**  $N=4, M=3, R=3, V=4$  (tetrahedron, standard d4)

**5.6.2**  $N=8, M=3, R=4, V=6$  (octahedron, standard d8)

**5.6.3**  $N=20, M=3, R=5, V=12$  (icosahedron, standard d20)

**5.6.4**  $N=6, M=4, R=3, V=8$  (cube, standard d6)

**5.6.5**  $N=12, M=5, R=3, V=20$  (dodecahedron, standard d12)

Some or all the corners on the die faces may be equivalent. By this we mean that they can't be told apart when looking at the face (barring mirroring or rotation) and are part of the same type of vertex. An example of this is the cube: Each face has four corners, but the vertices only have three corners meet. If the four corners of the faces all were different, some of the vertices would obviously also be different, and the faces would end up being aligned differently.

Equivalent faces must all belong to the same type of vertex (or else they could be told apart by their placement, and thus wouldn't be equivalent). Thus, if all the corners are equivalent, we can only have one type of vertex, and get the solutions shown above.

In the cases  $2 < M < 6$ , unless all the corners are equivalent, the only rotational symmetry is for a 180 degree rotation: There isn't room in a quadrangle for three-way symmetry, or in a pentagon for three-way or four-way symmetry. Thus, at most two corners can be equivalent (through 180 degree rotation or mirroring), though quadrangles and pentagons may have two such pairs. In other words, unless all the corners are equivalent, the corners must either be *singlets* or *doublets*.

For triangles, we find two possible combinations: 1 doublet plus 1 singlet, and 3 singlets. For quadrangles, we find three possible combinations: 2 doublets (either side-by-side or opposite corners), 1 doublet plus 2 singlets (with the doublet being two opposite corners), and 4 singlets.

For pentagons, we find two possible combinations: 2 doublets plus 1 singlet (with one doublet being the corners on either side of the singlet, and the other doublet being the remaining two corners), and 5 singlets. (You can't have 1 doublet plus 3 singlets: the doublet corners could not be placed symmetrically if the other three corners are different.) Let's examine a vertex consisting of only one type of corner. If this corner has two different adjacent corner types, then if the vertex has odd rank, those two different corners will be forced together in one of the adjacent vertices.

This observation can be expressed as follows:

**5.7** If a type of corner belongs to a vertex that contains no other types of corners, and the two corners adjacent to this corner type aren't a doublet pair, then the vertex must have even rank, or the adjacent vertices must contain both adjacent corners types.

A vertex can consist of more than one type of corners. Since a specific type of corner

only can be part of one type of vertex, the following must be true (since all corners must occur an equal number of times in the polyhedron):

**5.8** If a vertex consists of more than one type of corner, they must occur in proportion to their singlet/doublet status (different singlets equally, different doublets equally, a doublet twice as often as a singlet).

E.g., if a vertex is made up of a singlet and doublet types of corners, the vertex must contain 1 singlet corner plus 2 doublet corners, or multiples thereof. The rank of such a vertex must be a multiple of three. However, if a vertex only consists of one type of doublet corner, there is no requirement that the rank is even (see the example with the cube above). Similarly, if it consists of two types of doublet corners, 5.8 would be satisfied if it contained three corners of each type.

With the restrictions provided by 5.7 and 5.8, we can solve 5.6 for all the shapes described above. We only examine the cases where there's more than one type of vertex, since if there's only one, we just get the solutions 5.6.1 to 5.6.5 above.

Let's start with the triangular case of 1 doublet, 1 singlet. If the singlet and doublet corners are part of different types of vertices, 5.7 applies to the doublet vertices, which must have even rank. We get  $N = 2/(2/R_1 + 1/R_2 - 1/2)$ ,  $R_1$  even, which has the solutions:

**5.8.1**  $N$  even ( $N \geq 6$ ),  $M=3$ ,  $R_1=4$ ,  $V_1=N/2$ ,  $R_2=N/2$ ,  $V_2=2$  (double pyramid)

**5.8.2**  $N=12$ ,  $M=3$ ,  $R_1=6$ ,  $V_1=4$ ,  $R_2=3$ ,  $V_2=4$  (pyramids on tetrahedron faces)

**5.8.3**  $N=24$ ,  $M=3$ ,  $R_1=6$ ,  $V_1=8$ ,  $R_2=4$ ,  $V_2=6$  (pyramids on octahedron faces)

**5.8.4**  $N=60$ ,  $M=3$ ,  $R_1=6$ ,  $V_1=20$ ,  $R_2=5$ ,  $V_2=12$  (pyramids on dodecahedron faces)

**5.8.5**  $N=24$ ,  $M=3$ ,  $R_1=8$ ,  $V_1=6$ ,  $R_2=3$ ,  $V_2=8$  (pyramids on cube faces)

**5.8.6**  $N=60$ ,  $M=3$ ,  $R_1=10$ ,  $V_1=12$ ,  $R_2=3$ ,  $V_2=20$  (pyramids on icosahedron faces)

By "pyramids" in the above I mean any number of identical triangles meeting in a vertex. The standard d8 is a special case of 8.7.1 above, which only if  $N/2$  is even has opposing faces (and thus an "up" face when lying on a table).

In the triangular case of 3 singlets, let's first consider the case where two types of corners are part of the same type of vertex. We find that the equation becomes the same as for the case above, with  $R_1$  forced to be even by 5.8 rather than 5.7. Same equation = same solutions. We get nothing new.

If all three corners are part of different types of vertices, 5.7 requires that all the vertices have even rank.

We get  $N = 2/(1/R_1 + 1/R_2 + 1/R_3 - 1/2)$ , all  $R_i$  even.

We get the solutions:

**5.8.7**  $N=8,12,16,\dots$ ,  $M=3$ ,  $R_1=R_2=4$ ,  $V_1=V_2=N/4$ ,  $R_3=N/2$ ,  $V_3=2$   
(variant of 5.7.1 above)

**5.8.8**  $N=24$ ,  $M=3$ ,  $R_1=4$ ,  $V_1=4$ ,  $R_2=R_3=6$ ,  $V_2=V_3=4$  (variant of 5.7.3 above)

**5.8.9**  $N=48$ ,  $M=3$ ,  $R_1=4$ ,  $V_1=12$ ,  $R_2=6$ ,  $V_2=8$ ,  $R_3=8$ ,  $V_3=6$  (shape seen in crystals)

**5.8.10**  $N=120$ ,  $M=3$ ,  $R_1=4$ ,  $V_1=30$ ,  $R_2=6$ ,  $V_2=20$ ,  $R_3=10$ ,  $V_3=12$  (largest non-bipolar)

die)

By "variant" in the above, I mean that the shapes are topologically equivalent, the way that a cube elongated by pulling out opposite vertices is equivalent to an undeformed cube. Essentially, 5.8.7 and 5.8.8 are deforming versions of 5.8.1 and 5.8.3. You can build 5.8.9 and 5.8.10 by putting squeezed pyramids on the rhombic faces of 5.8.12 and 5.8.13 below, respectively.

We now turn to quadrangles. In the case 2 doublets, we get  $N = 2/(2/R_1 + 2/R_2 - 1)$ . We see that either  $R_1$  or  $R_2$  must be smaller than 4. Of the two possible ways of having 2 doublets, only the one where the doublets are opposite corners (i.e., rhombic faces) allow non-even ranks according to 5.7.

We get the solutions:

**5.8.11**  $N=6, M=4, R_1=3, V_1=4, R_2=3, V_2=4$  (elongated cube, a variant of 5.6.4)

**5.8.12**  $N=12, M=4, R_1=3, V_1=8, R_2=4, V_2=6$  (rhombic dodecahedron)

**5.8.13**  $N=30, M=4, R_1=3, V_1=20, R_2=5, V_2=12$  (the d30 sold in game shops)

The next case is 1 doublet, 2 singlets (kites). We first consider the case where the two singlets are part of the same type of vertex. We get the same equation as above (with the added limitation by 5.8 that  $R_2$  must be even), thus no new solutions. Next we consider the case where the doublet and one of the singlets are part of the same type of vertex. We get the equation  $N = 2/(3/R_1 + 1/R_2 - 1)$ , but we know that  $R_1$  must be a multiple of 3. We get this solution only:

**5.8.14**  $N$  even ( $N \geq 6$ ),  $M=4, R_1=3, V_1=N, R_2=N/2, V_2=2$  (**double cone made from kites**)

The cube and the d10 are both special cases of 5.8.14. Only if  $N/2$  is odd do these shapes have opposing faces (and thus an "up" face when lying on a flat surface).

If the doublet and both singlets all form their own vertices,

we get the equation  $N = 2/(2/R_1 + 1/R_2 + 1/R_3 - 1)$ , and we know from 5.7 that  $R_2$  must be even.

We get the solutions:

**5.8.15**  $N=12, M=4, R_1=4, V_1=6, R_2=R_3=3, V_2=V_3=4$  (a variant of 5.8.12)

**5.8.16**  $N=24, M=4, R_1=4, V_1=12, R_2=3, V_2=8, R_3=4, V_3=6$  (shape seen in crystals)

**5.8.17**  $N=60, M=4, R_1=4, V_1=30, R_2=3, V_2=20, R_3=5, V_3=12$

You can make 5.8.16 by replacing each face of a cube with four kites, and 5.8.17 in a similar manner from a dodecahedron.

The last case with quadrangles is where all the corners are singlets. If all belong to different corners, we get the equation  $N = 2/(1/R_1 + 1/R_2 + 1/R_3 + 1/R_4 - 1)$ . We know from 5.7 that all  $R_i$  must be even, and thus 4 or greater. No solutions can satisfy that. If two corner types are part of the same type of vertex, this vertex must have even rank according to 5.8. You thus can't have two double-singlet vertices (this would require that all  $R_i$  are even, which we have shown has no solution). If the two corner types belonging to the same vertex are adjacent, both singlet vertices must also have even rank (= no solutions), so the two corner

types must be opposite. This gives us the same equation that gave us 5.8.15, 5.8.16 and 5.8.17, and the same solutions. Similarly, in the case where three singlets are part of the same type of vertex, we reproduce solution 5.8.14. Thus, we find no new solutions by only having singlet corners.

We now turn to pentagons. We start with the case of 2 doublets and 1 singlet. If both doublets are part of the same type of vertex, we get  $N = 2/(4/R_1 + 1/R_2 - 3/2)$ .  $R_1$  must be even according to 5.8, and thus 4 or greater. No solutions can satisfy that. If one doublet and the singlet are part of the same type of vertex, we get the equation  $N = 2/(3/R_1 + 2/R_2 - 3/2)$ ,  $R_1$  a multiple of 3. We get one solution:

**5.8.18**  $N=12, M=5, R_1=3, V_1=12, R_2=3, V_2=8$  (a deform dodecahedron)

Next we have the case where both doublets and the singlet are part of different types of vertices. We get  $N = 2/(2/R_1 + 2/R_2 + 1/R_3 - 3/2)$ . From 5.7 we know that  $R_1$  and  $R_2$  both must be even, and thus 4 or greater, so we get no solutions.

We turn to the last case, namely all singlet corner types. If all but one are part of one type of vertex, we get  $N = 2/(4/R_1 + 1/R_2 - 3/2)$ , and  $R_1$  even. We have already shown that this has no solutions. Next we consider the case where 3 corner types are part of one type of vertex, and the remaining two both are part of another type. We get  $N = 2/(3/R_1 + 2/R_2 - 3/2)$ , with  $R_2$  even and  $R_1$  a multiple of 3 (from 8.8). This has no solutions. The next case has one type of vertex with three corner types, and two vertices with one corner type. We get  $N = 2/(3/R_1 + 1/R_2 + 1/R_3 - 3/2)$ ,  $R_1$  a multiple of 3. This has these solutions:

**5.8.19**  $N=12, M=5, R_1=3, V_1=12, R_2=R_3=3, V_2=V_3=4$  (another deform dodecahedron)

**5.8.20**  $N=24, M=5, R_1=3, V_1=24, R_2=3, V_2=8, R_3=4, V_3=6$

**5.8.21**  $N=60, M=5, R_1=3, V_1=60, R_2=3, V_2=20, R_3=5, V_3=12$

We can make 5.8.20 by placing sets of four pentagons on each face of a cube, turned a bit to make the corners interlace. 5.8.21 can be made in a similar manner from a dodecahedron. Even though the faces connect asymmetrically, these polyhedrons can be built with faces that have bilateral symmetry (if you don't consider how they connect).

The next case has two vertex types each with two corner types, and one vertex type with only one. We get  $N = 2/(2/R_1 + 2/R_2 + 1/R_3 - 3/2)$ ,  $R_1$  and  $R_2$  both even (from 5.8). This has no solutions. The next case has one vertex type with two corner types, and three vertex types with only one corner type. We get  $N = 2/(2/R_1 + 1/R_2 + 1/R_3 + 1/R_4 - 3/2)$ ,  $R_1$  even, and this also has no solutions. The last case has five different vertexes, each made from one corner type. 5.7 requires all of the vertices to have even rank, which cannot have a solution.

We now have gone through all the possibilities. There can be no proper dice except those mentioned in the solutions 5.1.1 to 5.8.21 above. We have made no requirements that the faces or edges must be non-curving, but it can be shown (by example) that all the solutions for  $M = 3, 4, \text{ or } 5$  can be made with non-curving faces and edges, as we have suggested in the text (and will show for some in illustrations below). In fact, I'm pretty sure (but offer no proof) that all of these can be made so all vertex points touch an enclosing sphere.

## 6. Dice Probabilities

The following formulae can be used to find the probability of rolling a sum S using N dice of M faces. The formulae provide the number of *permutations* that give the desired result; as in [17] to obtain the *probability* as an absolute number, divide by M<sup>N</sup>.

In the formulae,

{A:B} is the binomial coefficient

A!/B!(A-B)!, 0!=1, and SUM[i=0,j]( Xi ) is the sum of Xi for all values of i from 0 to j.

Number of permutations that give

$$\text{sum} = S: P(S, N, M) = \text{SUM}[i=0, j] ( (-1)^i * \{N:i\} * \{(S-i*M-1):(N-1)\} )$$

Where  $j = \text{Int}((S-N)/M)$ .

For large S the identity  $P(S) = P(Z)$ , where  $Z = N*(M+1)-S$ , can be used to simplify the calculation.

Number of permutations that give

$$\text{sum} = < T: P(< T, N, M) = \text{SUM}[i=0, j] ( (-1)^i * \{N:i\} * \{(T-i*M):N\} )$$

Where  $j = \text{Int}((T-N)/M)$ .

For large T the identity  $P(< T) = 1 - P(< Y)$ , where  $Y = N*(M+1)-T-1$ , can be used to simplify the calculation.

In some games you roll a pool of dice and have to count the number of dice that have a certain value or higher. For that and similar problems, the following formulae can be used to find the probability of various results. The notation is the same as above.

Number of permutations that have exactly I dice coming up with a value of exactly

$$J: P(I, J, N, M) = (M-1)^{(N-I)} * \{N:I\}$$

Number of permutations that have exactly I dice coming up with a value of J or higher:

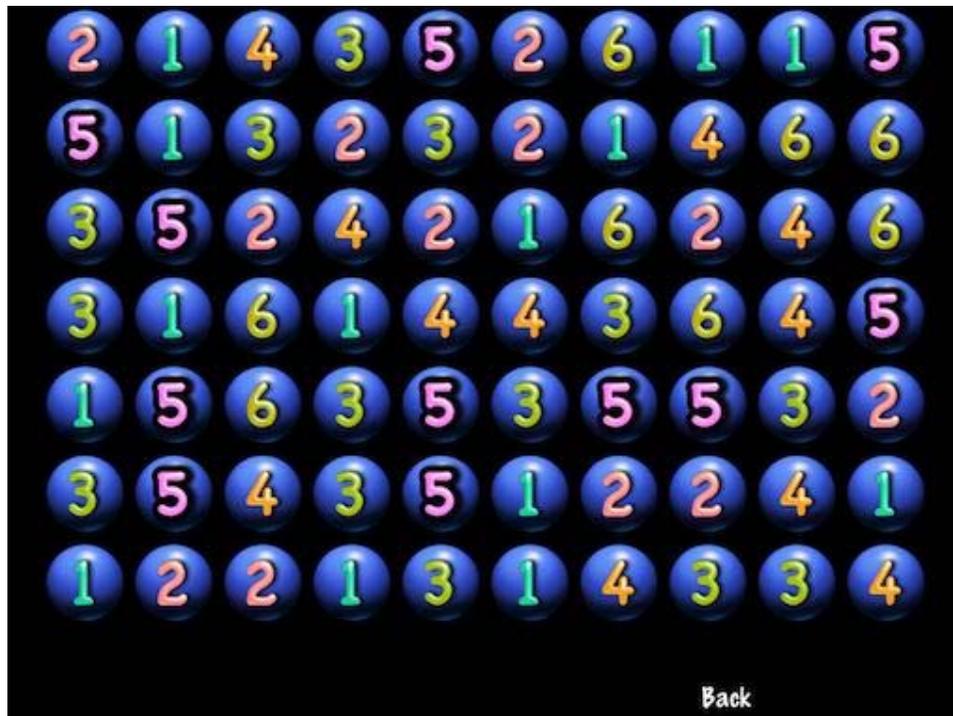
$$P(I, \geq J, N, M) = (J-1)^{(N-I)} * (M-J+1)^I * \{N:I\}$$

Number of permutations that have exactly I dice coming up with a value of J or less:

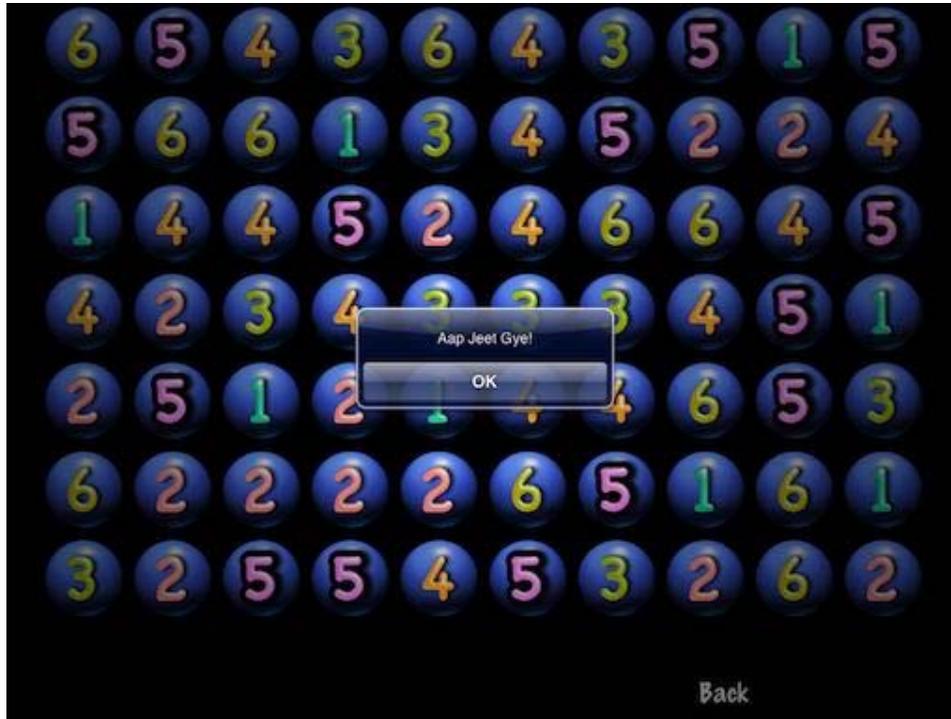
$$P(I, \leq J, N, M) = (M-J)^{(N-I)} * J^I * \{N:I\}$$

## 7. Diagrammatic representation of Dice Finder Game

Starting of game



After win the game —



## 8. Conclusion

Our proposed game improves Arithmetic power of students, having many fun and education and also helps children's and students to visit when they need more game with Math/Puzzle. Yet, several studies do not report enough data to draw significant conclusions. However, it is difficult to determine whether the game had the same impact on all of the students. Therefore, it is unclear whether or not higher achievement was attributed to playing the games or just engaging in more math tasks. In any case, the study suffers from the methodological flaw of conducting an evaluation of a pair of instances and attempting to generalize to the class; that is, games or classroom instruction. The authors concluded that further research is essential to figure out how to promote its use.

Results from these studies suggest that the mere presence of feedback, hints, or available help is insufficient for learning. One approach has been to train students in how to engage in effective help seeking (Salden, Alevan, & Renkl, 2007; Schwonke et al., 2007). Otherwise, in the literature on games, feedback, and help seeking, there is a noticeable gap in understanding how to motivate students to engage effectively in help seeking and to use provided feedback. The use of incentives for seeking help and feedback provides a relevant avenue for exploration.

## Acknowledgement

The authors are thankful to the Editor, International Journal of Mathematical Modelling & Computations for their editorial suggestions to make this paper in present form.

## References

- [1] Alan Holden, Shapes, Space and Symmetry, Columbia University Press, New York, 1971
- [2] Branko Grünbaum and G. C. Shephard, Spherical tilings with transitivity properties, in The Geometric Vein: The Coxeter Festschrift, C. Davis, B. Grünbaum and F. Shenk editors, Springer-Verlag, New York, 1982
- [3] Branko Grünbaum, On polyhedra in  $E^3$  having all faces congruent, *Bull. Research Council Israel*, **8F** 215-218(1960).
- [4] Editors; H. Behnke, F. Bachman, K. Fladt, and H. Kunle, translated by S. H. Gould, Fundamentals of Mathematics, Volume **11**, Geometry, MIT Press, Cambridge, 1974.
- [5] <http://en.wikipedia.org/wiki/Dice>
- [6] [http://en.wikipedia.org/wiki/Dice\\_notation](http://en.wikipedia.org/wiki/Dice_notation)
- [7] <http://homepage.ntlworld.com/dice-play/DiceStandard.htm>
- [8] <http://ia600402.us.archive.org/31/items/greekanthology05newyuoft/greekanthology05newyuoft.pdf>
- [9] <http://jnsilva.ludicum.org/HJT2k9/AlfonsoX.pdf>
- [10] <http://www.aeakybos.ch/Properties%20of%20Dice.pdf>
- [11] <http://www.aeakybos.ch/Properties%20of%22Dice.pdf>
- [12] <http://www.darkshire.net/~jhkim/rpg/whatis/>
- [13] <http://www.darkshire.net/~jhkim/rpg/whatis/tabletop.html>
- [14] [http://www.maa.org/editorial/mathgames/mathgames\\_05\\_16\\_05.html](http://www.maa.org/editorial/mathgames/mathgames_05_16_05.html)
- [15] <http://www.rpg.net/oracle/essays/rpgoverview.html>
- [16] <http://www.rpg.net/reviews/archive/13/13186.phtml>
- [17] <http://www-stat.stanford.edu/~cgates/PERSI/papers/fairdice.pdf>
- [18] Joseph B. Keller, The probability of heads, *Amer. Math. Monthly*, **191-197**, **93** (1986).
- [19] Martin Gardner's *Mathematical Games: The Entire Collection of his Scientific American Columns*
- [20] Owen Guillermo, Game Theory, Academic Press, USA
- [21] Peter Pearce, Structure in Nature is a Strategy for Design, MIT Press, Cambridge, 1978.
- [22] Sundeep Bhattacharya and Gaurav Srivastava, Game of Coordination for Bacterial Pattern Formation: A Finite Automata Modelling, *International Journal of Mathematical Modelling & Computations*, **3(4)**, 2013.