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A Fuzzy Model for Assessment Processes

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Abstract. The methods of assessing the individuals' performance usually applied in practice are based on principles of the bivalent logic (yes-no). However, fuzzy logic, due to its nature of including multiple values, offers a wider and richer field of resources for this purpose. In this paper we use principles of fuzzy logic in developing a new method for assessing the performance of groups of individuals participating in any human activity. For this, we represent each of the groups under assessment as a fuzzy subset of a set *U* of linguistic labels characterizing their members' performance and we use the centre of gravity defuzzification technique in converting the fuzzy data collected from the corresponding activity to a crisp number. The resulting structure provides a weighted assessment, in which the higher is an individual's performance the more its "contribution" to the corresponding group's performance. Thus, in contrast to the mean of the scores (i.e. numerical values of the performance) of all the group's members, which is connected to the mean group's performance, our method is connected somehow to the group's quality performance. Two real applications are also presented, related to the bridge players' performance and the students' problem solving skills respectively, illustrating the importance of our assessment method in practice.

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1. Introduction

There used tobe a tradition in science and engineering of turning to probability theory when one is faced with a problem in which uncertainty plays a significant role. This transition was justified when there were no alternative tools for dealing with the uncertainty. Today this is no longer the case. *Fuzzy logic*, which is based on fuzzy sets theory introduced by Zadeh [25]in 1965, provides a rich and meaningful addition to standard logic.

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A real test of the effectiveness of an approach to uncertainty is the capability to solve problems which involve different facets of uncertainty. Fuzzy logic has a much higher problem solving capability than standard probability theory. Most importantly, it opens the door to construction of mathematical solutions of computational problems which are stated in a natural language. The applications which may be generated from or adapted to fuzzy logic are wide-ranging and provide the opportunity for modelling under conditions which are inherently imprecisely defined, despite the concerns of classical logicians (e.g. see [8], Chapter 6 of [10], [13], [17-19], [20] and its relevant references, [21-24], etc).

The methods of assessing the individuals' performance usually applied in practice are based on principles of the bivalent logic (yes-no). However these methods are not probably the most suitable ones. In fact, fuzzy logic, due to its nature of including multiple values, offers a wider and richer field of resources for this purpose. In this paper we shall use fuzzy logic in developing a new general method for assessing the skills of groups of individuals participating in any human activity. The rest of the paper is organized as follows: In the next section we develop our new assessment method. In section three we present two real applications illustrating the importance of our method in practice. Finally the last section is devoted to conclusions and discussion for the future perspectives of research on this area. For general facts on fuzzy sets we refer freely to the book [10].

2. The Fuzzy Model

Let us consider a group, say H, of *n* individuals, where *n* is a positive integer, participating in a human activity (e.g. problem-solving, decision making, football match, a chess tournament, etc). Further, let *U* = {A, B, C, D, F} be a set of linguistic labels characterizing the individuals' performance with respect to the above activity, where A characterizes an excellent performance, B a very good, C a good, D a mediocre and F an unsatisfactory performance respectively. Obviously, the above characterizations are fuzzy depending on the user's personal criteria, which however must be compatible to the common logic, in order to be able to model the real situation in a worthy of credit way.

The classical way for assessing the total group's performance with respect to the corresponding activity is to express the levels of the individuals performance in numerical values and then to calculate the mean of their performance in terms of these values (*mean group's performance*).

Here, we shall use principles of fuzzy logic in developing an alternative method of assessment, according to which the higher is an individual's performance, the more its "contribution" to the group's total performance (*weighted group's performance*). For this, we are going to representH *as a fuzzy subset of U.* In fact, if n_A, n_B.n_C, n_D and n_F denote the number of the individuals of H that had demonstrated an excellent, very good, good, mediocre and unsatisfactory performance respectively at the game, we define the *membershipfunction (*We recall that the methods of choosing the membership function are usually empiric, based either on the common logic (as it happens in our case) or on the data of experiments made on a representative sample of the population that we study. The proper choice of the membership function is a necessary condition for developing a worthy of credit model of the corresponding situation using principles of fuzzy logic.)

$m:U \rightarrow [0, 1]$ as follows:

$$y = m(x) = \begin{cases} 1, & if \quad 80\% \ n < n_x \leq n \\ 0,75, & if \quad 50\% \ n < n_x \leq 80\% \ n \\ 0,5, & if \quad 20\% \ n < n_x \leq 50\% \ n \\ 0,25, & if \quad 1\% \ n < n_x \leq 20\% \ n \\ 0, & if \quad 0 \leq n_x \leq 1\% \ n \end{cases} \qquad (1)$$

for each x in *U*.

Then H can be written as a fuzzy subset of *U* in the form: H= {(x, m(x)): $x \in U$}.

In converting the fuzzy data collected from the corresponding activity we shall make use of the defuzzification technique known as the *method of the centre of gravity (COG)*.

According tothis method, the centre of gravityof the graph of the membership function involved provides an alternative measure of the system's performance [18]. The application of the COG method in our case is simple and evident and, in contrast to other defuzzification techniques in use, like the measures of uncertainty (for example see [20] and its relevant references, [23], etc), needs no complicated calculations in its final step.

The first step in applying the COG method is to correspond to each $x \in U$ an interval of values from a prefixed numerical distribution, which actually means that we replace *U* with a set of real intervals. Then, we construct the graph, say G,of the membership function *y=m(x)*. There is a commonly used in fuzzy logic approach to measure performance with the coordinates (x_c , y_c) of the *centre of gravity*, sayF_c, of the graph G,

which we can calculate using the following well-known from Mechanics formulas:

$$x_c = \frac{\iint_F x\,dxdy}{\iint_F dxdy}, \; y_c = \frac{\iint_F y\,dxdy}{\iint_F dxdy} \qquad (2)$$

In our case we characterize an individual's performance as unsatisfactory (F), if x $\in$ [0, 1), as mediocre (D), if x $\in$ [1, 2), as good (C), if x $\in$ [2, 3), as very good (B), if x $\in$ [3, 4) and as excellent (A), if x $\in$ [4, 5] respectively. In other words, if x $\in$ [0, 1), then y_1=m(x) = m(F), if x $\in$ [1, 2), then y_2=m(x)= m(D), etc.

Therefore, in our case the graph G of the membership function attached to H is the bar graph of Figure 1 consisting of five rectangles, say G_i, i=1,2,3, 4, 5 , whose sides lying on the Xaxis have length 1.

In this case $\iint_F dxdy$ is the area of G which is equal to $\sum_{i=1}^{5} y_i$

Also $\iint_F x\,dxdy = \sum_{i=1}^{5} \iint_{F_i} x\,dxdy = \sum_{i=1}^{5} \int_0^{y_i} dy \int_{i-1}^{i} x\,dx = \sum_{i=1}^{5} y_i \int_{i-1}^{i} x\,dx = \frac{1}{2}\sum_{i=1}^{5}(2i-1)y_i$, and

$$\iint_F y\,dxdy = \sum_{i=1}^{5} \iint_{F_i} y\,dxdy = \sum_{i=1}^{5} \int_0^{y_i} y\,dy \int_{i-1}^{i} dx = \sum_{i=1}^{n} \int_0^{y_i} y\,dy = \frac{1}{2}\sum_{i=1}^{n} y_i^{\,2} .$$

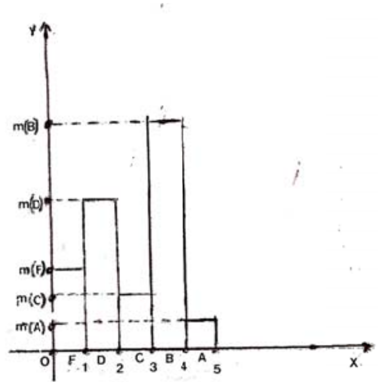

Figure 1: Bar graphical data representation

Therefore formulas (2) are transformed into the following form:

$$x_c = \frac{1}{2}\left(\frac{y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5}{y_1 + y_2 + y_3 + y_4 + y_5}\right),$$
$$y_c = \frac{1}{2}\left(\frac{y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2}{y_1 + y_2 + y_3 + y_4 + y_5}\right).$$

Normalizing our fuzzy data by dividing each $m(x)$, $x \in U$, with the sum of all membership degrees we can assume without loss of the generality that $y_1+y_2+y_3+y_4+y_5 = 1$ (We recall that, although *probabilities* and *fuzzy membership degrees* are both taking values in the interval [0, 1], they are *distinct* and *different to each other* notions. For example, the expression "The probability for Mary to be tall is 85%", means that, although Mary is either tall or low in stature, she is very possibly tall. On the contrary the expression "The membership degree of Mary to be tall is 0.85", simply means that Mary can be characterized as rather tall. A.characteristic difference (but not the only one) is that, while the sum of probabilities of all the singleton subsets (events) of *U* equals 1, this is not necessary to happen for the membership degrees.)Therefore we can write:

$$x_c = \frac{1}{2}\left(y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5\right), \qquad (3)$$
$$y_c = \frac{1}{2}\left(y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2\right)$$

with$y_i = \dfrac{m(x_i)}{\sum_{x\in U} m(x)}$.

But, $0 \leq (y_1-y_2)^2 = y_1^2+y_2^2-2y_1y_2$,therefore $y_1^2+y_2^2 \geq 2y_1y_2$, with the equality holding if, and only if, $y_1=y_2$. In the same way one finds that $y_1^2+y_3^2 \geq 2y_1y_3$, with the equality holding if, and only if, $y_1=y_3$ and so on. Hence it is easy to check that $(y_1+y_2+y_3+y_4+y_5)^2 \leq 5(y_1^2+y_2^2+y_3^2+y_4^2+y_5^2)$, with the equality holding if, and only if, $y_1=y_2=y_3=y_4=y_5$.
But $y_1+y_2+y_3+y_4+y_5 =1$, therefore
$1 \leq 5(y_1^2+y_2^2+y_3^2+y_4^2+y_5^2)$ (4),with the equality holding if, and only if, $y_1=y_2=y_3=y_4=y_5=\frac{1}{5}$

Then the first of formulas (3) gives that $x_c = \frac{5}{2}$. Further, combining the inequality (4) with the second of formulas (3), one finds that $1 \leq 10y_c$, or $y_c \geq \frac{1}{10}$. Therefore the unique minimum for y_c corresponds to the centre of gravity $F_m(\frac{5}{2}, \frac{1}{10})$.

The ideal case is when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. Then from formulas (3) we get that $x_c = \frac{9}{2}$ and $y_c = \frac{1}{2}$.Therefore the centre of gravity in this case is the point $F_i(\frac{9}{2}, \frac{1}{2})$.

On the other hand, in the worst case $y_1=1$ and $y_2=y_3=y_4= y_5=0$. Then by formulas (3), we find that the centre of gravity is the point $F_w(\frac{1}{2}, \frac{1}{2})$.

Therefore, the "area" where the centre of gravity $F_c$lies is represented by the triangle $F_wF_mF_i$ of Figure 2. Then from elementary geometric considerations it follows that the greater is the value of $x_c$the better is the corresponding group's performance.

Also, for two groups with the same $x_c \geq 2,5$, the group having the centre of gravity which is situated closer to $F_i$is the group with the higher y_c; and for two groups with the same $x_c<2.5$ the group having the centre of gravity which is situated farther to $F_w$is the group with the lower y_c. Based on the above considerations it is logical to formulate our criterion for comparing the groups' performances in the following form:

- *Among two or more groups the group with the higher $x_c$performs better.*
- *If two or more groups have the same $x_c \geq 2.5$, then the group with the higher $y_c$performs better.*
- *If two or more groups have the same $x_c < 2.5$, then the group with the lower y_c performs better.*

Notice that, similar techniques as above have been also used in [17], [24], etc.

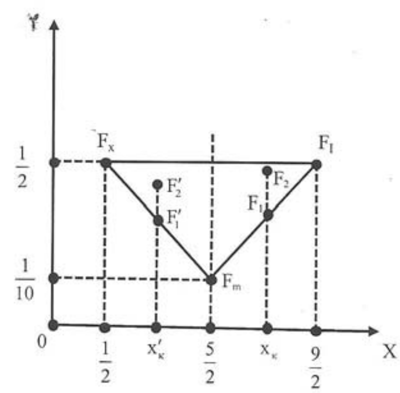

Figure 2: Graphical representation of the "area" of the centre of gravity

3. Real applications

In this section we shall present two real life applications illustrating in practice the importance of our results obtained in the previous section. The first of these applications concerns a new assessment method of the bridge players' performance, while the second one is related to the problem solvers' performance.

3.1 Anew assessment method of the bridge players' performance

C*ontract bridge* is a card game belonging to the family of trick-taking games. It occupies nowadays a position of great prestige being, together with chess, the only *mind sports* (i.e. games or skills where the mental component is more significant than the physical one) officially recognized by the International Olympic Committee. Millions of people play bridge worldwide in clubs, tournaments and championships, but also on line (e.g. [1]) and with friends at home, making it one of the world's most popular card games.

A match of bridge can be played either among *teams* (two or more) of four players (two partnerships), or among *pairs*. For a pairs event a minimum of three tables (6 pairs, 12 players) is needed, but it works better with more players. At the end of the match in the former case the result is the difference in *International Match Points (IMPs)* between the competing teams and then there is a further conversion, in which some fixed number of *Victory Points (VPs)* is appointed between the teams. It is worthy to notice that the table converting IMPs to VPs has been obtained through a rigorous mathematical manipulation [5].

On the contrary, the usual method of scoring in a pairs' competition is in *match points*. Each pair is awarded two match points for each pair who scored worse than them on each game's session (*hand*), and one match point for each pair who scored equally. The total number of match points scored by each pair over all the hands played is calculated and it is converted to a percentage.

However, IMPs can also used as a method of scoring in pair events. In this case the difference of each pair's IMPs is usually calculated with respect to the mean number of IMPs of all pairs.

For the fundamentals and the rules of bridge, as well as for the conventions usually played between the partners we refer to the famous book [9] of *Edgar Kaplan* (1925-1997), who was an American bridge player and one of the principal contributors to the game. Kaplan's book was translated in many languages and was reprinted many times since its first edition in 1964. There is also a fair amount of bridge-related information on the Internet, e.g. seeweb sites [2], [4], etc.

The *Hellenic Bridge Federation (HBF)* organizes, on a regular basis, *simultaneous* bridge tournaments (pair events) with pre-dealt boards, played by the local clubs in several cities of Greece. Each of these tournaments consists of six in total events, played in a particular day of the week (e.g. Wednesday), for six successive weeks. In each of these events there is a local scoring table (match points) for each participating club, as well as a central scoring table, based on the local results of all participating clubs, which are compared to each other. At the end of the tournament it is also formed a total scoring table in each club, for each player individually. In this table each player's score equals to the mean of the scores obtained by him/her in the five of the six in total events of the tournament. If a player has participated in all the events, then his/her worst score is dropped out. On the contrary, if he/she has participated in less than five events, his/her name is not included in this table and no possible extra bonuses are awarded to him/her.

In case of a pairs' competition with match points as the scoring method and according to the usual standards of contract bridge, one can characterize the players' performance, according to the percentage of success, say p, achieved by them, as follows:

Excellent (A), if $p > 65\%$.

Very good (B), if $55\% < p \leq 65\%$.

Good (C), if $48\% < p \leq 55\%$.

Mediocre (D), if $40\% \leq p \leq 48\%$.

Unsatisfactory (F), if $p < 40$ %. (In an analogous way one could characterize the players' performance in bridge games played with IMPs, with respect to the VPs gained.)

Our application presented here is related to the total scoring table of the players of a bridge club of the city of Patras, who participated in at least five of the six in total events of a simultaneous tournament organized by the HBF, which ended on February 19, 2014 (see results in [7]). Nine men and five women players are included in this table, who obtained the following scores. Men: 57.22%, 54.77%, 54.77%, 54.35%, 54.08%, 50.82 %, 50.82%, 49.61%, 47.82%. Women: 59.48%, 54.08%, 53.45%, 53.45%, 47.39%. These results give a mean percentage of approximately 52.696% for the men and 53.57% for the women players. Therefore the women demonstrated a slightly better mean performance than the men players, their difference being only 0.874%.

The above results are summarized in Table 1, the last column of which contains the corresponding membership degrees calculated with respect to the membership function defined by the formula (1) of the previous section. For example,

in the case of men players we have $n = 9$, 1% n = 0.09, 20% n = 1.8, 50% n = 4.5, 80% n =7.2, $n_A = n_F = 0$, $n_B = n_D = 1$, $n_C = 7$, which give that m(A) = m(B))= m(F) = 0, m(B) = m(D) = 0.25 and m(C)= 0.75

Men

% Scale	Performance	Number of players	*m(x)*
>65%	A	0	0
55-65%	B	1	0.25
48-55%	C	7	0.75
40-48%	D	1	0.25
<40%	F	0	0
Total		9	1.25

Women

% Scale	Performance	Number of players	*m(x)*
>65%	A	0	0
55-65%	B	1	0.25
48-55%	C	3	0.75
40-48%	D	1	0.25
<40%	F	0	0
Total		5	1.25

Table 1:Total scoring of the men and women players

Then, normalizing the membership degrees and applying formulas (3) of the previous section, we find that $x_c = \frac{1}{2*1.25}(3*0.25+5*0.75+7*0.25) = \frac{6.25}{2.5}$ =2.5 and y_c= $\frac{1}{2*(1,25)^2}[(0.25)^2+(0.75)^2+(0.25)^2] = \frac{0.6875}{3.125} = 0.22$ for both the men and women players..
Thus, according to our criterion (second case) stated in the previous section, and in contrast to their mean performance, the men demonstrated an identical weighted performance with the women players.

In concluding, our new assessment method of the bridge players' performance can be used as a complement of the usual scoring methods of the game (match points or IMPs) in cases where one wants to compare (for statistical or other reasons) the total performance of special groups of players (e.g. men and women, young and old players, players of two or more clubs participating in a big tournament, etc).

3.2 Assessing problem solving skills

The importance of Problem Solving (PS) has been realised for such a long time that in a direct or indirect way affects our daily lives for decades. It is generally accepted that PS is a complex phenomenon and no wonder there is no unique definition. However, the following definitions encompass most of the existing definitions: ForPolya [12], the pioneer in mathematical PS, "solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim that was not immediately understandable." According to Schoenfeld [14] "a problem is only a problem, if you don't know how to go about solving it. A problem that has no 'surprises' in store, and can be solved comfortably by routine or familiar procedures (no matter how difficult!), it is an exercise."Green and Gilhooly [6] state that "PS in all its manifestations is an activity that structures everyday life in a meaningful way." The authors add further that this activity draws together different components of cognition.

Therefore, the kind of problem will dictate the type of cognitive skill necessary to solve the problem: linguistic skills are used to read about a certain problem and debate about it, memory skills to recall prior knowledge and so on. Depending on the knowledge and thinking skills possessed by a problem solver, what could be a problem for one might not be a problem for some body else. Perhaps Martinez's [11] definition carries the modern message about PS: "PS can be defined simply as the pursuit of a goal when the path to that goal is uncertain. In other words, it's what you do when you don't know what you're doing."

Mathematics by its nature is a subject whereby PS forms its essence. In an earlier paper [22] we have examined the role of the problem in learning mathematics and we have attempted a review of the evolution of research on PS in mathematics education from its emergency as a self sufficient science at the 1960's until today.

While early work on PS focused mainly on analysing the PS process and on describing the proper heuristic strategies to be used in each of its stages, more recent investigations have focused mainly on solvers' behaviour and required attributes during the PS process; e.g. see Multidimensional PS Framework (MPSF) of Carlson & Bloom [3]. More comprehensive models for the PS process in general (not only for mathematics) were developed by Sternberg & Ben-Zeev [16], by Schoenfeld [15] (PS as a goal-oriented behaviour), etc. In earlier papers we have used basic elements of the finite Markov chains theory, as well as principles of the fuzzy logic (e.g. see [20] and its relevant references) in an effort to develop mathematical models for a better description and understanding of the PS process. Here, applying the results obtained in the previous section, we shall provide a new assessment method of the students' PS skills.

In our will to explore the effect of the use of computers as a tool in solving mathematical problems we performed during the academic year 2012-13 a classroom experiment, in which the subjects were students of the School of Technological Applications (prospective engineers) of the Graduate Technological Educational Institute (T. E. I.) of Western Greece attending the course "Higher Mathematics I"(The course involves Complex Numbers, Differential and Integral Calculus in one variable, Elementary Differential Equations and Linear Algebra.)of their first term of studies. The students, who had no previous computer experience apart from the basics learned in High School, where divided in two groups. In the control group (100 students) the lectures were performed in the classical way on the board, followed by a number of exercises and examples connecting mathematics with real world applications and problems. The students participated in solving these problems. The difference for the experimental group (90 students) was that part (about the 1/3) of the lectures and the exercises were performed in a computer laboratory. There the instructor used the suitable technological tools (computers, video projections, etc) to present the corresponding mathematical topics in a more "live" and attractive to students' way, while the students themselves, divided in small groups, used standard mathematical software to solve the problems with the help of computers.

At the end of the term all students participated in the final written examination for the assessment of their progress. The examination involved a number of general theoretical questions and exercises covering all the topics taught and three simplified real world problems (see Appendix) requiring mathematical modelling techniques for their solution (the time allowed was three hours). We marked the students' papers separately for the questions and exercises and separately for the problems. The results of this examination are summarized in Tables 2 {theoretical questions and exercises) and 3 (real world problems) , where the corresponding membership degrees were calculated in terms of the membership function (1), as it has been already shown in application 3.1.

Experimental group

% Scale	Performance	Number of students	*M(x)*
89-100	A	0	0
77-88	B	17	0.25
65-76	C	18	0.25
53-64	D	25	0.5
Less than 53	F	30	0.5
Total		90	1.5

Control group

% Scale	Performance	Number of students	*M(x)*
89-10	A	0	0
77-88	B	18	0.25
65-76	C	20	0.25
53-64	D	30	0.5
Less than 53	F	32	0.5
Total		100	1.5

Table 2: Theoretical questions and exercises

Experimental group

% Scale	Performance	Number of Students	*M(x)*
89-100	A	3	0.25
77-88	B	21	0.5
65-76	C	28	0.5
53-64	D	22	0.5
Less than 53	F	16	0.25
Total		90	2

Control group

% Scale	Performance	Number of students	*M(x)*
89-100	A	1	0
77-88	B	10	0.25
65-76	C	37	0.5
53-64	D	31	0.5
Less than 53	F	21	0.5
Total		100	1.75

Table 3: Real world problems

Normalizing the membership degrees and applying formulas (3), it is easy to check from Table 2 that the two groups demonstrated identical performance with respect to their students' replies to the theoretical questions and their solutions of the exercises. In fact, for both groups we find that

$$x_c = \frac{1}{2*1.5}(0.5+3*0.5+5*0.25+7*0.25) = \frac{5}{3} \approx 1.667$$

and y_c=

$$\frac{1}{2*(1,5)^2}[(0.5)^2+(0.5)^2+(0.25)^2+(0.25)^2] = \frac{0.569}{4.5} \approx 0.126$$

(third case of our criterion).
Similarly from Table 3 we find

$$x_c = \frac{1}{2*2}(0.25+3*0.5+5*0.5+7*0.5+9*0.25) = \frac{10}{4} = 2.5$$ for the experimental and

$$x_c = \frac{1}{2*1.75}(0.5+3*0.5+5*0.5+7*0.25) = \frac{6.25}{3.5} \approx 1.786$$

for the control group, which means that the experimental group demonstrated a better performance with respect to the solution of the real problems than the control group.

Further, it is worthy to notice that the experimental group's performance was significantly better with respect to the solution of the real world problems than with respect to the theoretical questions and exercises (1.667<2.5). The same happened with the control group, but in a much smaller degree (1.667<1.786).

In concluding, the results of our experiment provide a strong indication that the use of computers as a tool for PS enhances the students' abilities in solving real world mathematical problems.

4. Conclusions and discussion

In the present paper we developed a new method for assessing the total performance of groups of individuals participating in any kind of human activity. In developing the above method we represented each of the groups under assessment as a fuzzy subset of a set *U* of linguistic labels characterizing their members' performance and we used the COG defuzzification technique in converting the fuzzy data collected from the corresponding activity to a crisp number. According to the above assessment method the higher is an individual's performance the more its "contribution" to the corresponding group's total performance (weighted performance). Thus, in contrast to the mean of the scores (i.e. numerical values of the performance) of all the group's members, which is connected to the mean group's performance, our method is connected somehow to the group's *quality performance*. As a result, when the above two different assessment methods are used in comparing the performance of two or more groups of individuals, the results obtained may differ to each other in cases where there are small differences in the groups' performance (e.g. see our bridge application). Two real applications were also presented, related to the bridge players' performance and the students' problem solving skills respectively, illustrating the importance of our assessment method in practice.
Our future plans for further research on the subject aim at applying our new assessment method in more real situations of bridge matches (including also games played with IMPs) and problem solving (not only mathematical) applications in order to get statistically safer and more solid conclusions about its applicability and usefulness. In a wider spectre, since our method is actually a general assessment method, it could be interesting to be applied in more sectors of the human activity, including other competitive games (e.g. more card games, chess, backgammon, etc), collective and individual sports, human cognition and learning, Artificial Intelligence, Biomedical Sciences, Management and Economics, etc.

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Appendix: The problems given for solution to students in our classroom experiments

Problem 1: We want to construct a channel to run water by folding the two edges of an orthogonal metallic leaf having sides of length 20cm and 32 cm, in such a way that they will be perpendicular to the other parts of the leaf. Assuming that the flow of the water is constant, how we can run the maximum possible quantity of the water?
Remark: The correct solution is obtained by folding the edges of the longer side of the leaf. Some students solved the problem by folding the edges of the other side and failed to realize that their solution was wrong (*validation of the model*).
Problem 2: Let us correspond to each letter the number showing its order into the alphabet (A=1, B=2, C=3 etc). Let us correspond also to each word consisting of 4 letters a 2X2 matrix in the obvious way; e.g. the matrix $\begin{bmatrix} 19 & 15 \\ 13 & 5 \end{bmatrix}$ corresponds to the word SOME. Using the matrix E= $\begin{bmatrix} 8 & 5 \\ 11 & 7 \end{bmatrix}$ as an encoding matrix how you could send the message LATE in the form of a camouflaged matrix to a receiver knowing the above process and how he (she) could decode your message?
Problem 3: The population of a country is increased proportionally. If the population is doubled in 50 years, in how many years it will be tripled?