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Balancing and Sequencing Versus Only Balancing in Mixed Model U-Line Assembly Systems: an Economic Analysis

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Abstract. With the growth in customers demand diversification, mixed-model U-lines (MMUL) have acquired increasing importance in the area of assembly systems. There are generally two different approaches in the literature for balancing such systems. Some researchers believe that since the types of models can be very diverse, a balancing approach without simultaneously sequencing of models will not yield an optimum configuration. On the other hand, another group of researchers point to the high cost of balancing systems and prefer to do it only one time regardless of the models sequences.In this paper, we aim to compare these two approaches by introducing an economic indicator. To do so, two models as representatives of the two different viewpoints are taken from the literature. To check the validity of this methodology, it is implemented by Lingo 11.0, for small scale, and GA, for a large scale. The obtained results indicate that, from the proposed economic indicator point of view, mixed-model U-lines balancing and sequencing (MMUL/BS) is preferred to its counterpart, mixed-model U-lines balancing (MMULB).This paper offers economic guidelines for managers to choose between only balancing and implementing it by sequencing at the same time.

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Keywords: Mixed model U-lines; Assembly line balancing; Assembly line balancing and Sequencing; Cost index; Genetic algorithm.

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1. Introduction

Assembly line balancing is the process of assigning a set of tasks to an ordered sequence of stations in such a fashion that some efficiency measures; like cycle time,

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number of stations and so on are optimized subject to the precedence communication between the tasks.In other words, a method of balancing has been modernized to achieve a fuller integration between design and management of assembly lines. There are two sorts of problems related to assemble line:

- A direction that some performance measures are optimized subject to precedence relationship among tasks, called as line balancing (LB).
- The problem of selecting the order or sequence in which different models will be produced, called as model sequencing (MS).

Balance is determined by grouping tasks into the station while going on ward through a precedence diagram.
The succession is found by sorting machines and equipments that performance is optimized in accordance with client requirement.
An assembly line balancing problem (ALBP) consists of distributing the total workload for manufacturing any unit of the products to be assembled among the work stations along the line. In some other universe, the decision problem of optimally balancing the assembly work among the stations with respect to some objectives is known as ALBP.Most of the techniques employed to solve ALBP require the appointment of each task to a single workstation and, consequently, the yield rate is determined by the longest task time.
When there is just one example of a product that is being set up along the crease, the resulting problem is addressed the simple assembly line balancing problem (SALBP) fall into four main categories Boysen et al. [5].

- SALBP-1: The problem of assigning operations to stations is formulated with the target of minimizing the number of stations used so as to take on a target cycle time
- SALBP-2: Against the first type, the objective function for this type of problems is to minimize the cycle time for a committed number of workstations W.
- SALBP-E: When neither of the cycle time C nor the number of stations W is given, the objective function is maximizing the line efficiency E where E is calculated from the following equation:

$$E = \frac{tsum}{(W.C)}$$

- SALBP-F: In this kind of problem, both of the cycle time C and number of stations Wis given and the objective is to find a feasible balance.

Extending the basic problem by considering sequence-dependent is called as SDALBP.
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As an outcome of just-in-time (JIT) execution, manufacturers aim to achieve continuously reduced inventory, faster throughput, increased productivity, decreased defects and better control product quality by getting rid of all wastes in their production systems. Thestraight line cannot completely confirm the acceptance of JIT principles to manufacturing especially in the use of multi-skilled operators. So, manufactures have replaced it with U-line. In U-assembly line workers are gathered along the inside of the curve and communication is easier than on the length of

a straight line. Assemblers can see each process; what is occurring and how fast; and one individual can execute multiple operations.Also, workstations along the "line" are capable to make multiple product designs simultaneously, causing the facility as a whole more flexible. The u-shaped problem with addressing straight line involve of several subjects:

(1) Extend of communication
(2) The multi-skill operator
(3) The simplification of re-balancing line

A mixed model assembly line is a type of production line which is used to assemble a variety of product models with a certain level of similarity in operational characteristics. This variety leads workload variance among other problems resulting in low efficiency and line stops.
Many producers are shifting their production lines from a single product or batch production to mixed-model production (Figure 1). In mixed-model production, different products or models are brought out along the same job with the models interspersed throughout a production sequence.This kind of occupations is generally employed to develop the flexibility to adjust to the changes in market demand.As mixed-model production is performed on U-lines we have a mixed-model U-seam.

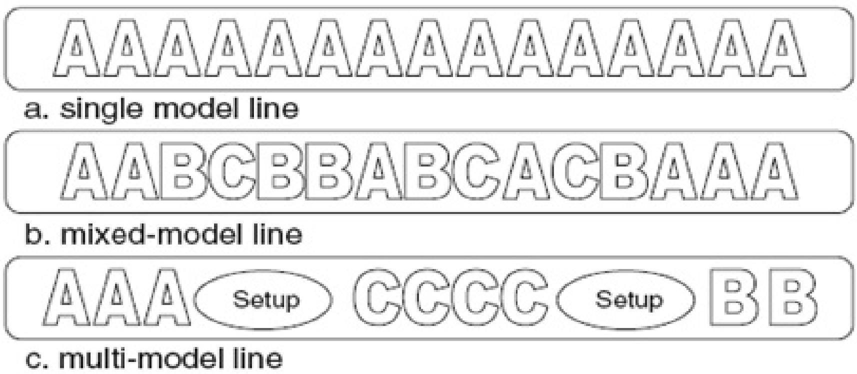

Figure 1. Different types of assembly lines ([17])

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature followed by the problem. In Section 3 the proposed mathematical model is explained. In Section 4, we discussed on solving methodology. Computational results are shown in section 5. Finally, concluding remarks and some possible future research directions are given in Section 6.

2. Literature Review

Salveson[35] discussed on Assembly Line Balancing (ALB) problem for the foremost time.Karp[16] stated ALB problem is NP-hard. Becker and Scholl[4]
provided the review of exact and heuristic solution procedures for simple assembly line balancing (SALB), mixed-model assembly lines (MMAL) etc. Kriengkorakot and Pianthong[20]introduced the U-Line assembly line balancing problem (UALBP) and noticed that it was more complex than the straight line assembly problem.
Sparling and Miltenburg[36]mentioned on MMULB to minimize the number of stations and absolute deviation of workload (ADW) and considered assigning tasks

to an ordered sequence of places for the foremost time. Karabat and Sayn[15] attempted to minimize total cycle time and reduce optimally maximum sub-cycle time by using genetic algorithm.Battini et al. [3] paid attention to the un-paced assembly lines to minimize idle time and overload time across workstations and solved it based on branch and bound methodologies and realized that balancing MMAL system does not secure a right part of workload between stations.Kazemi et al.[17] ran a MMULB problem by a two stage genetic algorithm (GA) with duplicated task. In 2012, they minimized the crossover workstations and ADW, maximized the efficiency of assembly line and ran it with GA by considering the operator movement time. Boysen et al.[5]classified assembly line balancing as: rebalancing, cost synergies, Sequence-dependent time increments, feeder lines, material supply, parallel working places, processing alternatives, disassembly line balancing, test beds and flexible solution procedures. Akpnar and Bayhan[2]targeted at downplaying the number of workstations and maximizing smoothness between and within stations.

Merengo et al. [24] conducted MMUL/BS in manual field to aim to (1) minimize the rate of incomplete jobs, (2) reduce WIP. He classified Balancing objective is at: (1) minimizing the number of stations, (2) reducing WIP, (3) facilitating line manager. Sequencing objective is: (1) minimizing the rate of incomplete tasks (in continuous lines), (2) minimizing the probability of blocking and starvation events (in un-paced lines), (3) reducing WIP and (4) having a constant material flow. There are several interesting and relevant MMUL/BS papers which are minimized ADW with the aim of minimizing and deviation of part production quantities in a JIT environment by using genetic algorithm (GA) ([25]), minimizing the number of stations by simulated annealing algorithm (SA)([12], [13]; [30]), with the application of SA ([14]), symbiotic evolutionary algorithm ([18]), end symbiotic evolutionary algorithm (EEA)([19]). Hwang and Katayama[9] tried to reduce the number of stationsand variety of workloads. In [10], they developed and ran it by genetic algorithm for assembling sequencing (ASGA). For generating chromosome, ASGA apply priority-based multi-chromosome (PMC). Mosadegh et al. [27]aimed to reduce the total utility work of new case studies, employed a new Mixed-Integer Linear Programming (MILP) and ran it with SA. At last, they compared their result to the Co-evolutionary Genetic Algorithm (Co-GA) and Hill Climbing (HC) by considering the quality solution and the time of achieving the best solution. Buxey [6]explained the benefit of parallel workstation such as balance efficiency and reaching high production rates. Hamzadayi and Yildiz[7] solved MMUL/BS problem to aim to minimize the number of workstations and smooth the workload between and within workstations by presenting a priorityBased Genetic Algorithm (PGA).In 2013, they showed ADW is an insufficient criterion to assess the performance of the solutions.Rahimi-Vahed and Mirzaei[32] tried to minimize total utility work, total production rate variation, and total setup cost and their method were a hybrid multi-objective algorithm based on shuffled frog-leaping algorithm (SFLA) and bacteria optimization (BO). Miltinburg (2002) used GA and concluded that sequencing is depending on balancing. Several researchers have developed MMUL/BS([11])with the application of Ant Colony Optimization (ACO)([21]), Collaborative Ant Colony Optimization (CACO) ([1]), the colonial competitive algorithm CCA and its modified version (MCCA) ([22]).zcan et al. [29]performed MMUL/BS with stochastic task time and ran it with GA. They said if assembly operations are executed with more advanced machines are highly qualified operators, the task time is constant. Machine breakdowns, lack of trained operators, complex tasks and so on lead to stochastic task time.Tseng and Tang [37]discussed on connectors equipped with assembly engineering information and evaluate the ASP

and ALB. Rashid et al. [33]surveyed mentioned issue. Riezebos[34]focused on the capacity balancing problem in a synchronous manufacturing system.Manavizadeh et al. [23] proposed a multi objective model. Their model minimized the work overload, the cycle times and the wastage in each station. For solving, they suggested a new heuristic algorithm that can improve the result from the initial solution. Also, their model can help the decision maker for choosing about the assembly lines shape.
In this section, it is shown that some authors believe that since the types of models can be very diverse, MMULB will not yield an optimum configuration. On the other hand, another group of researchers point to the high cost of balancing systems and prefer to do it only one time regardless of the models sequences. In this paper, we aim to compare these two approaches by presenting an economic indicator. To do so, two models as representatives of the two different viewpoints are taken from the literature.Moreover, due to the comparison principle, the methods and examples of the base papers are applied.

3. Model Description

Many producers are shifting their production lines from a single product or batch production to mixed-model production.In this field, some articles was suggested sequencing after balancing nevertheless, another group prefer to carry out balancing and sequencing simultaneously. Firstly, in this research, we proposed a cost index. By it, manufacturer easily realizes which one is better?. Secondary, two represented models will be run by using our index. Due to the close the reality, we select Rabbani et al.[31], for MMULB, and Hamzadayi and Yildiz[7], for MMUL/BS in order to extend and implement indicators on. Since the high cost of balancing and sequencing, this approach is very significant.The following presents the nomenclature of the model.

3.1 *Set of Indices*

Q	Number of sequences with	$q = 1 \dots Q$
S	Number of stations with	$s = 1 \dots S$
M	Number of models with	$m = 1 \dots M$
K	Number of tasks with	$k = 1 \dots K$
Z	Number of machines with	$z = 1 \dots Z$

3.2 *Notations*

3.2.1 Set of parameters

Proposed cost index:

C_{vw}	The cost of WIP
C_{zl}	The cost of unit longest relocation for machine z
$t(S_k)$	The sum of time of the k-th station
t_s	The time of station
L	The length of line that machine move
U_s	The utilization of station
V_w	Workload (or WIP)
IZ_z	The price of machine z

CS The cost of the new station
XO_s The level of skill of operator in station s
CZ_z The cost of machine z in unit time
W_q The completion time of all stations in cycle time
q_k The sequence of models for task s

MMUL/B:

K Total number of different assembly tasks
M Number of models to be assembled on the line
$S_{\max}$ An upper bound for number of workstations
t_{km} Processing time of taskk for product model m
L The length of line that machine move
U_s The utilization of station
V_w Workload (or WIP)
IZ_z The price of machine z
CS The cost of the new station
XO_s The level of skill of operator in station ss
CO The cost of operating in unit time
CZ_z The cost of machine z in unit time
W_q The completion time of all stations in cycle time
q_k The sequence of models for task k
β An extra coefficient for crossover workstation s
C Cycle time of the line
δ An extra time added to the workstations considering the operators travel times

P Set of precedence relationships in the combined (original)precedence diagram $(r, k) \in P$. A precedence relationship; assembly task r must be done before task k

$U_{m,s}$ Utilization of product model m on workstation s
a_m Average utilization of model m for total workstations$s$

MMUL/B

K Total number of tasks are performed in a set of workstation
$(k = 1 \ldots K)$
S Number of workstations utilized on the mixed model U-line
$(s = 1 \ldots S)$
P The planning horizon has a fixed length
M Number of different models produced on MMUL $(m = 1 \ldots M)$
R The length of the model sequence for one $MPS = \sum_{m=1}^{M} d_m$
Q U-line cycle $(q = 1, 2 \ldots Q)$
C Cycle time: P/R
t_{km} The requiring time to perform task k on the model m m
XF_s Set of tasks in workstation s located on the front of the U-line
XB_s Set of tasks in workstation s located on the back of the U-line
f_s^q The model produced on the front of the workstations at the cycle in the sequence
b_s^q The model produced on the back of workstations at the cycle in the sequence

3.2.2 Set of variables

Proposed cost index:

$$\mathrm{XZ}_{s,z} = \begin{cases} \textit{If the station s need to machine z} & 1 \\ o.w & 0 \end{cases}$$

MMUL/B:

$$\mathrm{X}_{ks(Y_{ks})} = \begin{cases} \textit{If task k is assigned to station s from the original (phantom) diagram} & 1 \\ o.w & 0 \end{cases}$$

$$\mathrm{Z}_s = \begin{cases} \textit{if station s is utilized and regular} & 1 \\ o.w & 0 \end{cases}$$

$$\mathrm{S}_s = \begin{cases} \textit{if station s is utilized and crossover} & 1 \\ o.w & 0 \end{cases}$$

MMUL/BS:

$$\mathrm{RPs}= \begin{cases} \textit{if workstation k is required more than one } (RWs > 1\ ) & 1 \\ o.w & 0 \end{cases}$$

he U-lines considered in this paper are adapted fromRabbani et al. [31] with some-ofassumption, but we add cost index to their model which explains under the following assumptions:

- A combined precedence diagrams are employed for different precedence diagrams of models.
  It is worth to say the predecessors and successors of one task are previously allocated.
- By considering an extra time, try to cover travel time of operator and set up time is ignored.
- Common tasks for different models must be apportioned to the individual post.
- The model arrives to each crossover workstation, in front of the line is only similar the one arrives to that workstation in the back of the line.
- An extra coefficient has been assigned for minimizing of crossover stations.
- This model only considers a balancing problem.
- There are no restrictions on allocated of tasks to any station.

$$Max\frac{1}{M}(\sum_{m=1}^{M}\frac{\sum_{k=1}^{K}t_{km}}{[\sum_{s=1}^{s_{max}}(Z_s+S_s)+\beta\times\sum_{s=1}^{s_{max}}S_s]\times c}+1-(\frac{1}{\sum_{s=1}^{s_{max}}(Z_s+S_s)}\times\sum_{s=1}^{s_{max}}(U_{m,s}-a_m)^2)^{\frac{1}{2}}])$$

$$Min(\sum_{q=1}^{Q}\sum_{s=1}^{S}\sum_{z=1}^{Z}(XZ_{z,s}\times CZ_z\times W_q)) \qquad (C_{z,L}\times L)$$

$$+(\sum_{s=1}^{S}C+\sum_{s=1}^{S}\sum_{z=1}^{Z}(XZ_{z,s}\times IZ_z))+(\sum_{q=1}^{Q}\sum_{s=1}^{S}(XO_s\times CO\times W_q))$$

$$\sum_{s=1}^{S_{max}}(X_{ks}+Y_{ks})=1 \qquad \forall k(1)$$

$$\sum_{s=1}^{S_{max}} S\times(X_{rs}-X_{ks})\leqslant 0 \qquad \forall(r,k)\in P(2)$$

$$\sum_{s=1}^{S_{max}} S\times(Y_{ks}-Y_{rs}]\leqslant 0$$

$$\sum_{k=1}^{K} t_{km}\times(X_{ks}+Y_{ks})\leqslant C\times Z_s+(C-\delta)\times S_s \qquad \forall s,m \tag{3}$$

$$\begin{cases} \sum_{(k=1})^{K}(X_{ks}-K\times O_s\geqslant 1-K) \qquad \forall s,k \\ \sum_{(k=1})^{K}(X_{ks}-K\times O_s\leqslant 0) \\ \sum_{(k=1})^{K}(Y_{ks}-K\times g_s\geqslant 1-K) \\ \sum_{(k=1})^{K}(Y_{ks}-K\times g_s\leqslant 0 \\ O_s+g_s-2\times S_s-Z_s=0 \qquad \forall k \end{cases} \tag{4}$$

$$U_{(m,s)}=\frac{1}{C}\sum_{(k=1)}^{K} t_{km}\times(X_{ks}+Y_{ks} \tag{5}$$

$$a_m=\frac{1}{(\sum_{s=1}^{S_{max}}(Z_s+S_s)}\times\sum_{s=1}^{S_{max}} U_{(m,s)} \qquad \forall m \tag{6}$$

$$Z_s,P_s,X_{ks},Y_{ks},O_s,g_s\in\{0,1\} \qquad \forall k,s \tag{7}$$

The objective function (Eq. (1)) consists of two segments. The first branch of section one maximizes line efficiency, bearing in mind coefficient β 0, it can be fulfilled that. In fact the solution task of the model is to minimize the total number of workstations while the minimization of crossover workstations is done in the scope that the total number of stations (for the both types of situations).In the

other word, the model seeks to swap a crossover workstation (CW) by only one regular workstation (RW) without the total number of stations change.For this purpose, the model by two amounts of β (= 0 and a value less than 1) is solved, therefore the number of CWs and RWs comparisons, with observing different two optimum patterns deduced it, this is a resolution that has less number of CWs with the same number of places.The second piece of section one of the objective function minimizes variation of workload; in fact, it has been taken to minimize variation of work load that could outcome caused by different model mixes in crossover stations.In the second section of the proposed objective function, the total costs assigned with the number of installations, human source, reallocation machines and stationset up are minimized.
Constraint (1) states that every task must be allocated to just one station and only one time using the original or apparition diagram. For satisfying precedence relation between assembly tasks set of constraints (2) presented. Constraint (3) guarantees that the total act time of each model at each regular or crossover station subtracted from the cycle time. In Eq. (4), a set of workstation constraints integrated to the model for calculating the type of regular or crossover station.In constraint (5), utilization of product model mon workstations is calculated and average utilization of model m for total workstations is determined by Eq. (6). Constraint (7) represented that variables are binary.
Model of Balancing and sequencing by considering cost index:
The U-lines considered in this paperare adapted from Hamzadayi and Yildiz [7]with the same assumption, but we add cost index to their model which explains under the following assumptions:

- The travel times of the operators between stations and setup times are ignored.
- Product models having alike production epithets are produced on the same U-shaped production lines.
- The completion times of tasks may vary from one model to another and can be equal to zero.
  Common tasks among different models exist.
- A combined precedence diagrams are employed for different precedence diagrams of models.
- Fixed set-up rate and equally equipped stations are considered.
- Task completion times are constant and independent from each others.

$$Min \qquad Z = S+Cb+Cw+\sum_{q=1}^{Q}\sum_{s=1}^{S}\sum_{z=1}^{Z}(XZ_{s,z}\times CZ_z\times W_q)+\sum_{q=1}^{Q}\sum_{s=1}^{S}(XO_s\times CO\times W_q)+(C_{zL})$$

$$+\sum_{s=1}^{S}CS+\sum_{s=1}^{S}\sum_{z=1}^{Z}(XZ_{s,z}\times IZ_z)+(C_{Vw}\times\sqrt{\frac{\sum_{s=1}^{S}((\frac{\sum_{j\in s_k}t_j}{\max_{s=1}^{S}\sum_{j\in s_k}t_j})-\frac{\sum_{s=1}^{S}U_S}{S}))^2}{S}})$$

$$RWs=[\frac{t_{k(f_s^q+b_s^q)}}{c}k\in(XF_s+XB_s);q=1,\ldots,Q] \qquad (s=1,\ldots,S) \qquad (8)$$

$$MAXRP = RW_s + CP \qquad (s = 1, \ldots, S) \qquad (9)$$

$$S = \sum_{s=1}^{S}[1 + RP_s(RP - 1)] \qquad (10)$$

$$ST_s = C \times [1 + RP_s(RP - 1)] \qquad (s = 1, \ldots, S) \qquad (11)$$

$$S_{sq} = (ST_s - W_{sq}) \qquad (s = 1, \ldots, S) \qquad (q = 1, \ldots, Q) \qquad (12)$$

$$KI_s = \sum_{q=1}^{Q} S_{sq} \qquad (s = 1, \ldots, S) \qquad (13)$$

$$Cb = \frac{R}{(S(R-1)} \sum_{s=1}^{S}\sum_{q=1}^{Q}(\frac{S_{sq}}{KI_s} - \frac{1}{R})^2 \qquad (14)$$

$$RI_q = \sum_{s=1}^{S} S_s q \qquad (q = 1, \ldots, Q) \qquad (15)$$

$$Cw = \frac{S}{(R(S-1)} \sum_{q=1}^{Q}\sum_{s=1}^{S} \frac{S_{sq}}{RI_q}\frac{1}{S})^2 \qquad (16)$$

The objective function minimizes the number of workstations and smoothing the workload between-within workstations at the conclusion of all cycle and the total costs assigned with the number of installations, human source, and reallocation machines and set up station and workload (WIP) ([28]).
Eq. (8) when all the U-line bikes are held into consideration it shows that minimum how many replications must be made out to the workstationS for being capable to manage the task having maximal task time under the cycle time restriction.When use of a workstation increases, it can be copied. In Eq. (9), to capture the alternative paralleling situation, the decision maker may define the extra replicas of workstation, CP.It sets aside the decision makers to exercise below the different scenarios.Thus, MAXRP means a workstation can be replicated up to an upper limit on the utmost number of replications. Eq. (10) shows the entire number of workstations on the assembly line is calculated by the summation of the number of replicas of workstations.The workload capacity of the workstation S Eq. (11).In Eq. (12) Ssqshows the unused time of workstation S at the cycle q (the deviation between the capacitance of the workstation and its workload means the idle time of a workstation).In Eq. (13) KIs shows the total idle time at the conclusion of all cycles in the workstation S.Eq.(14) is second term (Cb) that its goal is smoothing

the workload of workstations between cycles (it means if the unused time is spread across all cycles as equally as possible for each workstation.The value of function Cb varies between a upper limit of 1 and a minimum of 0, the first one occurs when the total idle time on a workstation at the conclusion of all cycles equal to just one cycle idle time, and the second one falls out when the idle times of a workstation in the each cycle are equal to each other).The RIq shows the total idle time of all workstations at the cycle q (Eq. (15)). In Eq. (16) Cw are the third term with goals at smoothing the work load of all workstations within each cycle (i.e., the idle time is scattered across all workstations as equally as possible at each cycle. The value of function Cw varies between a upper limit of 1 and a minimum of 0, the first one occur when the total idle times on all workstations at any cycle equal to only one workstations idle time, and the second one happen when the idle times of each workstation at any cycle are equal to each other). The proposed performance measure may differ depending on the line balance and the model sequence([7], [8]).

3.3 *Illustrateexample*

In this subdivision, we create a new test problem (see Figure 2 and 3 and Table 1)and are solved by our proposed model using Lingo 11.0 software to show the comparison the exact solution of each type of problem in small scale.

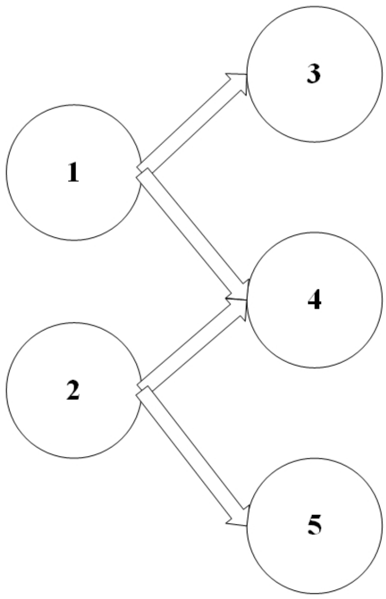

Figure 2. Precedence Diagram

We solved this example by our integer linear programming for the MMULBP and MMUL/BS separately using Lingo 11.0 software and the optimal objective function value (OFV) found 6133.330 and 5202.271 respectively.

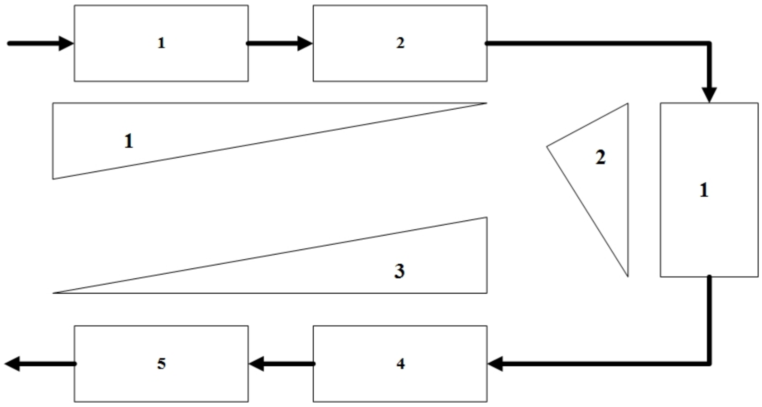

Figure 3. Optimal balance for the mixed-model U-line production system

Table 1. Completion task time for test problem

Task		**Processing times**	
	Model 1	Model 2	Model 3
1	0.5	0	1
2	0.4	0.8	1.2
3	0.2	0	0
4	04	0.5	0.6
5	0	0.5	0.5

4. Solution Methodology

Both base papers,Rabbani et al.[31]and Hamzadayi and Yildiz [7], were applied Genetic algorithm as the solution methodology, hence this algorithm is used to compare both models in principle.

4.1 *Genetic representation on MMULB*

In this paper, two different chromosomes are considered: (1) chromosomes line, (2) Chromosome matrix. Each chromosome is generated randomly for initial population. By following the flowchart proposed in Figure 4, the Genetic algorithm represented on MMULB can understand easily.

4.1.1 Fitness function

In the proposed genetic algorithm, the fitness function value of each solution is figured by the objective function value of the related solution

4.1.2 Crossover

In this technique, two integers between 1 and length of chromosomes are arbitrarily generated. Then two selected parents swap their genes between these two numbers (see Figure5).

4.1.3 Mutation

For generating the chromosome, firstly, two numbers between 1 and length of the chromosome that is arbitrary, create randomly. We assume that the smallest number is a and the largest one is b. Then $b - a + 1$integers between 1 and 10 are randomly produced and replaced in the chromosome. Each of offspring produced by crossover operation is mutated with probability of mutation rate and then goes

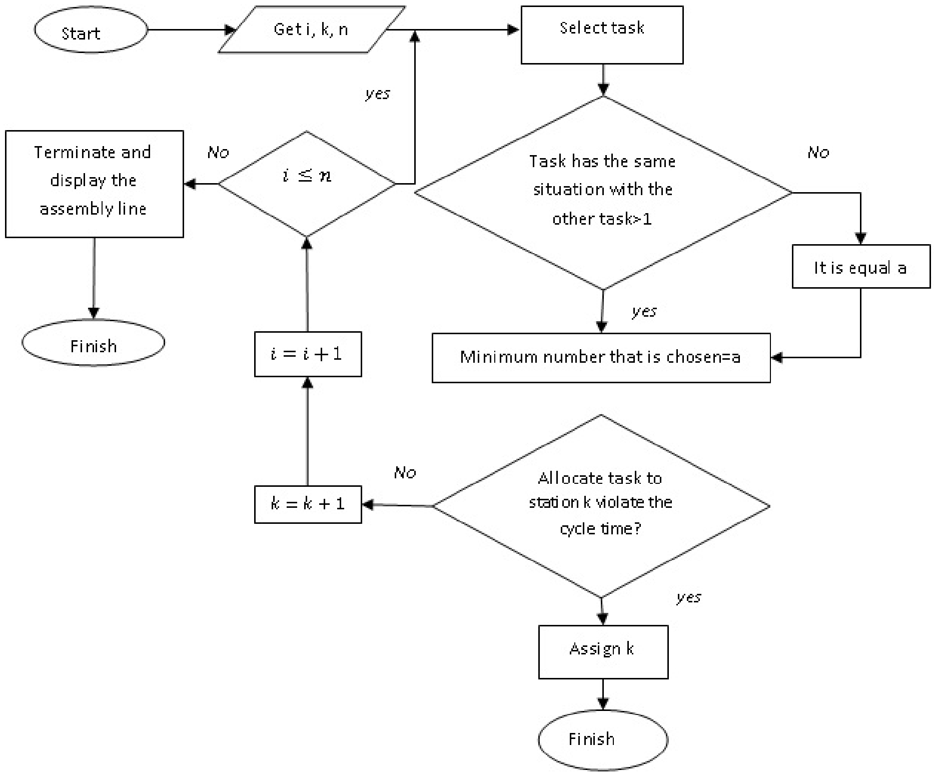

Figure 4. Flow-chart of the proposed GA for MMUL

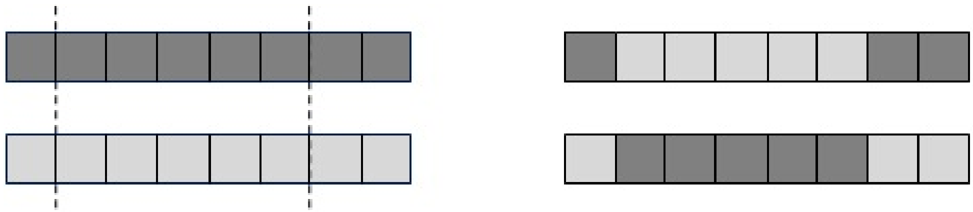

Figure 5. Crossover operator of MMULB

to the next generation.

4.1.4 Selection

The most common sampling mechanism to select solutions (chromosomes) from sampling space is Roulette Wheel selection technique which is a stochastic sampling approach. In this method, for each chromosome, a selection probability proportional to its fitness function is:

$$Prob(Solution(j)) = \frac{Fitness(Solution(j)}{\sum_{j=1}^{posize} Fitness(Solution(j)}$$

4.2 *Genetic representationofMMUL/BS*

We see two types chromosome for this problem: (1) for the PR is a Chromosomes line, (2) for the XZ is a Chromosome matrix. Each chromosome is generated randomly for initial population.

4.2.1 Fitness Function

In this paper, we study the fitness function of Hamzadayi and Yildiz[7] and added a cost index that is fully delineated in the section of modeling.

4.2.2 Crossover

We apply the Cross operator (single point) for the both chromosomes like showing in Figure 6.

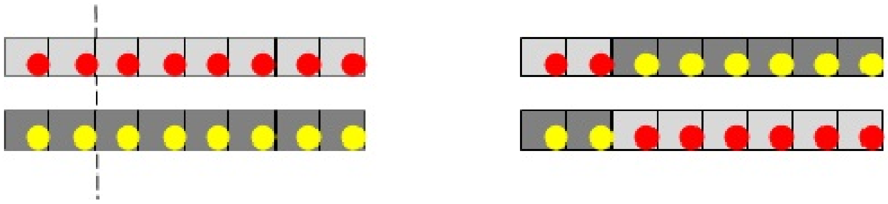

Figure 6. Crossover operator of MMUL/BS

4.2.3 Mutation

We apply two methods for this measure. In the first two points of the chromosome is randomly selectedand then we swapping them together. Another method is that one point of a chromosome is randomly selectedand then the number reversion randomly.
In the last step these populations are merged. This is as initial population for the next iteration. The run is stopped when the responsehasno significant changes. By tracking the flowchart proposed in Figure 7, the GA represented in MMUL/BS can find easily.

5. Computational Results

In order to carry on our computational experiments, test problems with mixed-model data are required.We set out with the test problems given in the benchmark data set of Thomopoulos test problem (with 19 tasks), Kim test problem (with 61 tasks) and Arcus test problem (with 111 tasks) and generate the additional information that is required to try out the various approaches presented in the old regions.It is worth to note that the examples are standard, hence, the results can be more confident and we can extend them to new issues.These are solved using GA.We solved MMULB with =0.5 and =0.5. The other parameters and result are shown asfollows in Table(2).

Table 2. The parameters of model in GA

Type of model	Number of population	Crossover rate	Mutation rate
MMULB	50	0.9	0.3
MMUL/BS	50	0.9	0.3

If the result of running of GA (Table 3) is compared with the previous response in Rabbani et al. [31], our MMULB results would be better than it. The result

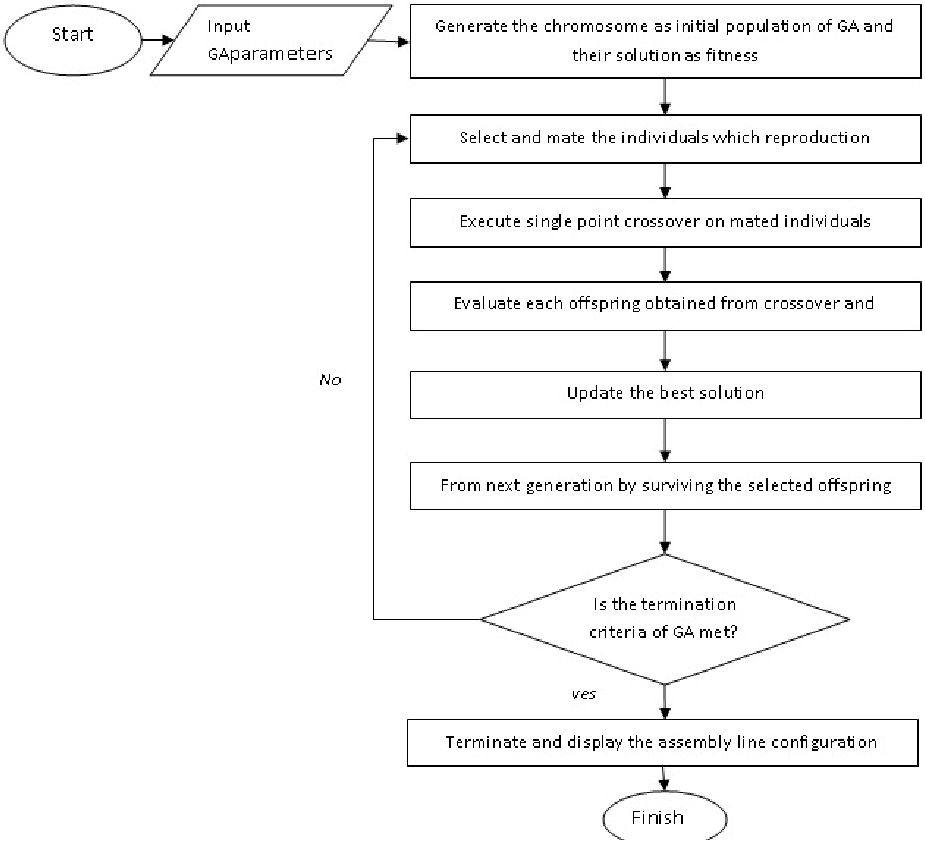

Figure 7. Flowchart of GA for MMUL/BS

of MMUL/BS and its cost index is better than MMULB. Also, the obtained results of Lingospecify that, from the proposed economic indicator point of view, MMUL/BS is preferred to its counterpart, MMULB.

6. Conclusion and Future Research Directions

Many manufacturing is changing their production lines from a single product or batch production to mixed model production, due to the variety customers demand. Moreover, with rising up JIT and Lean production, firms have realized that the U-shape line met their plans, more than a straight line. Hence, nowadays, factories design their assemble plan as mixed model U-line. According to the literature review, the authors surveyed two types of problems: (1) balancing and sequencing simultaneously, (2) only balancing.

As mentioned previously, there are many articles conducted on two above types of problems. In this paper, we propose an economic index due to determine which one has the less cost?. Hence, we define our index commensurate with the types of problem and add them to the objectives of Rabbani et al. [31] for MMULB problem and Hamzadayi and Yildiz [7]for MMUL/BS problem respectively.

Referable to the comparison principle, the methods and examples of the base papers are given. For small scale, we run our problems that it took less than one second with the result of MMUL/BS is better than MMULB. The proposed algorithm has been programmed in the MATLAB (R2012a) software. A computational study is completed to investigate the performance of the GA. We run them three

Table 3. The Results of model in GA

Type of model	Test problem (Cycle time)	CPU time	Objective	Economic Index
MMULB	Thomopoulos(10)	407.25	1.6815	0.58
MMULB	Kim(12)	759	1.5732	0.60
MMULB	Kim(16)	568	1.6248	0.68
MMULB	Kim(24)	427	1.7027	0.73
MMULB	Arcus(8500)	2051	1.7954	0.77
MMULB	Arcus(11,378)	2000.87	1.8108	0.85
MMULB	Arcus(17,067)	2726.42	1.8425	0.83
MMULB	Arcus(34000)	2561.03	1.8226	0.80
MMULB	Thomopoulos(10)	104.5	1.4523	0.50
MMUL/BS	Kim(12)	373.2	1.5284	0.55
MMUL/BS	Kim(16)	356	1.5924	0.61
MMUL/BS	Kim(24)	349.8	1.6743	0.69
MMUL/BS	Arcus(8500)	1945.9	1.6961	0.73
MMUL/BS	Arcus(11,378)	1879.25	1.7568	0.79
MMUL/BS	Arcus(17,067)	2022.64	1.8302	0.82
MMUL/BS	Arcus(34000)	1984.1	1.8140	0.80

times based on Thomopoulos test problem (with 19 tasks), Kim test problem (with 61 tasks) and Arcus test problem (with 111 tasks). It is worth to note that the examples are standard, hence, the consequences can be more confident and we can expand them to new subjects. The GA was run successfully within negligible CPU times. It is worth that the MMUL/BS is better than MMULB with the aspect of cost.

For future research, we recommended to extend this paper in set up time and the time of moving of operators between the station directions. Also, our approach can be extended to the case of inventory cost and the storage issues [26], [36]. In addition, the other meta-heuristic approaches and efficient heuristic approaches may be developed to solve the problem in future research.

References

[1] S. Agrawal and M. Tiwari, A collaborative ant colony algorithm to stochastic mixed-model U-shaped disassembly line balancing and sequencing problem, International Journal of Production Research, **46** (2008) 1405-1429.

[2] S. Akpnar and G. M. Bayhan, A hybrid genetic algorithm for mixed model assembly line balancing problem with parallel workstations and zoning constraints, Engineering Applications of Artificial Intelligence, **24** (2011) 449-457.

[3] D. Battini, M. Faccio, A. Persona and F. Sgarbossa, Balancingsequencing procedure for a mixed model assembly system in case of finite buffer capacity, The International Journal of Advanced Manufacturing Technology, **44** (2009) 345-359.

[4] C. Becker and A. Scholl, A survey on problems and methods in generalized assembly line balancing, European Journal of Operational Research, **168** (2006) 694-715.

[5] N. Boysen, M. Fliedner and A. Scholl, A classification of assembly line balancing problems, European Journal of Operational Research, **183** (2007) 674-693.

[6] G. Buxey, Assembly line balancing with multiple stations,Management science, **20** (1974) 1010-1021.

[7] A. Hamzadayi and G. Yildiz, A genetic algorithm based approach for simultaneously balancing and sequencing of mixed-model U-lines with parallel workstations and zoning constraints, Computers Industrial Engineering, **62** (2012) 206-215.

[8] A. Hamzadayi and G. Yildiz, A simulated annealing algorithm based approach for balancing and sequencing of mixed-model U-lines, Computers Industrial Engineering, **66** (2013) 1070-1084.

[9] R. Hwang, H. Katayama, A multi-decision genetic approach for workload balancing of mixed-model U-shaped assembly line systems, International Journal of Production Research, **47** (2009) 3797-3822.

[10] R. Hwang and H. Katayama, Integrated procedure of balancing and sequencing for mixed-model as-

sembly lines: a multi-objective evolutionary approach,International Journal of Production Research, **48** (2010) 6417-6441.

[11] M. Kamal Uddin, M.Cavia Soto and J. L. Martinez Lastra, An integrated approach to mixed-model assembly line balancing and sequencing, Assembly Automation, **30** (2010) 164-172.

[12] Y. Kara, U. Ozcan and A. Peker, An approach for balancing and sequencing mixed-model JIT U-lines,The International Journal of Advanced Manufacturing Technology, **32** (2007a) 1218-1231.

[13] Y. Kara, U. Ozcan and A. Peker, Balancing and sequencing mixed-model just-in-time U-lines with multiple objectives,Applied Mathematics and Computation, **184** (2007b) 566-588.

[14] Y. Kara, Line balancing and model sequencing to reduce work overload in mixed-model U-line production environments, Engineering Optimization, **40** (2008) 669-684.

[15] S. Karabat, S. Sayn, Assembly line balancing in a mixed-model sequencing environment with synchronous transfers, European Journal of Operational Research, **149** (2003) 417-429.

[16] R. M. Karp, Reducibility among combinatorial problems: Springer,(1972)

[17] S. M. Kazemi, R. Ghodsi, M. Rabbani and R. Tavakkoli-Moghaddam, A novel two-stage genetic algorithm for a mixed-model U-line balancing problem with duplicated tasks,The International Journal of Advanced Manufacturing Technology, **55** (2011) 1111-1122.

[18] Y. K. Kim, J. Y. Kim and Y. Kim, A coevolutionary algorithm for balancing and sequencing in mixed model assembly lines, Applied Intelligence, **13** (2000) 247-258.

[19] Y. K. Kim, J. Y. Kim and Y. Kim, An endosymbiotic evolutionary algorithm for the integration of balancing and sequencing in mixed-model U-lines, European Journal of Operational Research, **168** (2006) 838-852.

[20] N. Kriengkorakot and N.Pianthong, The U-line Assembly Line Balancing Problem, KKU Engineering Journal, **34** (2007) 267-274.

[21] [I. Kucukkoc and D. Z. Zhang, Simultaneous balancing and sequencing of mixed-model parallel two-sided assembly lines, International Journal of Production Research, **52** (2014) 3665-3687.

[22] K. Lian, C. Zhang, L. Gao and X. Shao, A modified colonial competitive algorithm for the mixed-model U-line balancing and sequencing problem,International Journal of Production Research, **50** (2012) 5117-5131.

[23] N. Manavizadeh, M. Rabbani and F. Radmehr, A new multi-objective approach in order to balancingand sequencing U-shaped mixed model assembly line problem:a proposed heuristic algorithm,Int J Adv Manuf Technol, **36** (2015) 485-501.

[24] C. Merengo, F. Nava and A. Pozzetti, Balancing and sequencing manual mixed-model assembly lines,International Journal of Production Research, **37** (1999) 2835-2860.

[25] J. Miltenburg, Balancing and scheduling mixed-model U-shaped production lines, International Journal of Flexible Manufacturing Systems, **14** (2002) 119-151.

[26] V. K. Mishra, Production Inventory Model for Deteriorating Items with Shortages and Salvage Value Under Reverse Logistics,International Journal of Mathematical Modelling Computations,**(2)** **2** (2012) 99-110.

[27] H. Mosadegh, M. Zandieh and S. F. Ghomi, Simultaneous solving of balancing and sequencing problems with station-dependent assembly times for mixed-model assembly lines,Applied Soft Computing, **12** (2012) 1359-1370.

[28] [A. Nourmohammadi, M. Zandieh, R. Tavakkoli-Moghaddam, An imperialist competitive algorithm for multi-objective U-type assembly line design, Journal of Computational Science, **4** (2013) 393-400.

[29] U. zcan, H. eriolu, H.Gken and B. Toklu, Balancing and sequencing of parallel mixed-model assembly lines,International Journal of Production Research, **48** (2010) 5089-5113.

[30] U. zcan, T. Kellegz and B. Toklu, A genetic algorithm for the stochastic mixed-model U-line balancing and sequencing problem,International Journal of Production Research, **49** (2011) 1605-1626.

[31] M. Rabbani, S. M. Kazemi and N. Manavizadeh, Mixed model U-line balancing type-1 problem: A new approach, Journal of Manufacturing Systems, **31** (2012) 131-138.

[32] A. Rahimi-Vahed and A. H. Mirzaei, A hybrid multi-objective shuffled frog-leaping algorithm for a mixed-model assembly line sequencing problem, Computers Industrial Engineering, **53** (2007) 642-666.

[33] M. F. F. Rashid, W. Hutabarat and A. Tiwari, A review on assembly sequence planning and assembly line balancing optimisation using soft computing approaches, The International Journal of Advanced Manufacturing Technology, **59** (2012) 335-349.

[34] J. Riezebos, Order sequencing and capacity balancing in synchronous manufacturing, International Journal of Production Research, **49** (2011) 531-552.

[35] M. E. Salveson, The assembly line balancing problem,Journal of Industrial Engineering, **6** (1955) 18-25.

[36] S. Singh, R. Dube and S. R. Singh, Production Model with Selling Price Dependent Demand and Partial Backlogging Under Inflation , International Journal of Mathematical Modelling Computations, **1** (2011) 01-07.

[37] D. Sparling and J. Miltenburg, The mixed-model U-line balancing problem,International Journal of Production Research, **36** (1998) 485-501.

[38] H. E. Tseng, C.-E.Tang, A sequential consideration for assembly sequence planning and assembly line balancing using the connector concept,International Journal of Production Research, **44** (2006) 97-116.