International Journal of
Mathematical Modelling & Computations
Vol. 06, No. 01, Winter 2016, 19- 27

Fixed Point Theorem of Kannan-Type Mappings in Generalized Fuzzy Metric Spaces

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Abstract. Binayak et al in [Binayak S. Choudhury and Kirshnapada Das, *Fixed point of generalized Kannan-type mappings in generalized menger spaces*, Commun. Korean. Math. Soc., (4) 24 (2009) 529-537] proved a fixed point of generalized Kannan type-mappings in generalized Menger spaces. In this paper we extend generalized Kannan-type mappings in generalized fuzzy metric spaces. Then we prove a fixed point theorem of this kind of mapping in generalized fuzzy metric spaces. Finally we present an example of our main result.

Received: 13 July 2015, Revised: 6 December 2015, Accepted: 7 january 2016.

Keywords: Generalized Fuzzy metric space, Kannan-type mappings, Generalized Kannan-type mappings, Fixed point.

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1. Introduction

A generalization of the concept of metric space was obtained by Branciari [2]. Also, in the same work, Banach contraction mapping theorem in generalized metric spaces was established. Olaleru, et al [12] generalize some results on coupled fixed point theorems of generalized ϕ-mappings in cone metric spaces. In 1942, Menger [11] introduced the notion of probabilistic metric space as a generalization of metric space. Such a probabilistic generalization of metric spaces appears to be well adapted for the investigation of physical quantities and physiological thresholds.

Kannan-type mappings was first discovered by R. Kannan in 1968 [9]. These mappings are important in metric fixed point theory for several reasons. First,

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the Banach contraction mapping theorem is for continuous maps, but Kannan-type mapping does not need necessarily be continuous. Another reason is their connection with metric completeness. In fact, Subrahmanyam in [**?**] proved that a metric space (X, d) is complete if and only if every Kannan-type mapping on X has a fixed point. Note that a Banach contraction mapping may have a fixed point in metric space which is not complete.

Binayak et al [1] introduced generalized Menger space as a generalization of Menger space as well as a probabilistic generalization of generalized metric space and, then, proved a fixed point of generalized Kannan type-mappings in generalized Menger spaces. Kramosil and Michalek [10] gave a notion of fuzzy metric space which could be considered as a reformulation, in the fuzzy context, of the notion of probabilistic metric space. Menger [11] and others have intensively studied the fixed point theory of these spaces (see [6–8]).

In this work generalized fuzzy metric space is considered as an extension of fuzzy metric space. Then, we prove a fixed point of generalized Kannan-type mappings in generalized fuzzy metric space. Then we give some example of our results.

2. Preliminaries

Before giving details of the proof of the main result, we begin by recalling some basic definitions and preliminaries of our notations.

Definition 2.1 *Let X be a nonempty set and $d : X^2 \to \mathbb{R}^+$ a mapping such that for all $x, y, z, u \in X$,*

(i) $d(x, y) \geqslant 0$, $d(x, y) = 0 \Leftrightarrow x = y$,
(ii) $d(x, y) = d(y, x)$,
(iii) $d(x, y) \leqslant d(x, z) + d(z, u) + d(u, y)$.

Then (X, d) is called a generalized metric space.

Obviously, every metric space is a generalized metric space, but the reverse is not true [4].

Example 2.2 Let $X = \{a, b, c, e\}$ and $d : X^2 \to \mathbb{R}^+$ be defined by
$d(a, b) = 0.25$, $d(a, c) = d(b, c) = 0.1$, $d(a, e) = d(b, e) = d(c, e) = 0.2$ and

$$d(x, x) = 0, \qquad d(x, y) = d(y, x) \qquad x, y \in X.$$

Then it is easy to check that (X, d) is a generalized metric space, whereas it is not a metric space because

$$d(a, b) = 0.25 > d(a, c) + d(b, c) = 0.2.$$

In 1965, the concept of fuzzy set was introduced by Zadeh [15] which laid the foundation of fuzzy mathematics. The concept of fuzzy metric space has been introduced in different ways by some authors(see i.e. [5, 10]). In 1975, Kramosil and Michalek [10] introduced the idea of fuzzy metric space, which created a path for additional expantion of analysis in such spaces. Further, George and Veeramani [11] modified this structure [10] with a view to obtain a Hausdoroff topology on it. Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory, mathematical programming and etc.

Definition 2.3 *A function B from a set X to the closed unit interval* $[0, 1]$ *in* $\mathbb{R}$ *is called a fuzzy set in X. And for every $x \in X$, $B(x)$ is a membership grade of x*

in B*. The set* $\{x \in X : B(x) > 0\}$ *is called the support of* B*.*

Kramosil and Michalek [10] and George and Veeramani [7] considered the notation of fuzzy metric space by useing continuous t-norms. This paper makes use of Kramosil and Michalek definition of fuzzy metric space.

Definition 2.4 *An operation* $* : \Pi_{i=1}^{n}[0,1] \to [0,1]$ *is an* n*-th order continuous* t*-norm if* $([0,1],*)$ *is a commutative topological monoid with unit 1 such that*

$$a_1 * a_2 * a_3 * ... * a_n \leqslant b_1 * b_2 * b_3 * ... * b_n$$

whenever $a_i \leqslant b_i$ *for each* $a_i, b_i \in [0,1]$, $\quad i = 1,2,...,n$.

Definition 2.5 *A 3-tuple* $(X, M, *)$ *is said to be a fuzzy metric space, if* X *is a nonempty set,* $*$ *is a continuous t-norm and* M *is a fuzzy set on* $X^2 \times [0,\infty)$ *where the following statments hold for all* $x, y, z \in X$ *and* $s, t > 0$*:*

(i) $M(x,y,0) = 0$,
(ii) $M(x,y,t) = 1$ *for all* $t > 0$ *if and only if* $x = y$,
(iii) $M(x,y,t) = M(y,x,t)$,
(iv) $M(x,y,t) * M(y,z,s) \leqslant M(x,z,t+s)$,
(v) $M(x,y,.) : [0,\infty) \to [0,1]$ *is left-continuous.*

If $(X, M, *)$ is a fuzzy metric space we say that $(M,*)$ (or simply M) is a fuzzy metric on X.

Example 2.6 Let (X,d) be a metric space and $a * b = ab$ for all $a, b \in [0,1]$

$$M_d(x,y,t) = \frac{t}{t + d(x,y)} \qquad x, y \in X, \quad t > 0,$$

$M_d(x,y,t)$ is a fuzzy metric space. M_d is called the standard fuzzy metric induced by the metric d. $M(x,y,t)$ can be thought of as the degree of nearness between x and y with respect to t.

Fuzzy metric space can be generalized in a way similar to metric space. In [3] Chugh and Kumar introduced the generalized fuzzy metric space $(X, M, *)$ as follows.

Definition 2.7 *A 3-tuple* $(X, M, *)$ *is said to be a generalized fuzzy metric space if* X *is a nonempty set,* $*$ *is a 3-rd continuous* t*-norm and* M *is a fuzzy set on* $X^2 \times [0,\infty)$*, where the following statments hold for all* $x, y, z \in X$ *and* $r, s, t > 0$*:*

(i) $M(x,y,0) = 0$,
(ii) $M(x,y,t) = 1$ *for all* $t > 0$ *if and only if* $x = y$,
(iii) $M(x,y,t) = M(y,x,t)$,
(iv) $M(x,y,t) * M(y,z,s) * M(z,u,r) \leqslant M(x,u,t+s+r)$,
(v) $M(x,y,.) : [0,\infty) \to [0,1]$ *is left-continuous.*

It is easy to check that every fuzzy metric space is a generalized fuzzy metric space. Next lemma shows that every generilized metric space is a generalized fuzzy metric space.

Lemma 2.8 *Every generalized metric space* (X,d) *is a generalized fuzzy metric space.*

Proof Let $*$ be the 3-rd order continuous minimum t-norm given by

$$x * y * z = \min\{x,y,z\}, \qquad x, y, z \in X.$$

Let $M(x,y,t) = H(t - d(x,y))$ such that

$$H(s) = \begin{cases} 0 & s \leqslant 0 \\ 1 & s > 0 \end{cases},$$

where $x, y \in X$ and $t > 0$. Obviously, the conditions (i), (ii), (iii) and (v) of generalized fuzzy metric space hold. If the condition (iv) does not hold, then there exist $x, y, z, u \in X$ and $r, s, t > 0$ such that,

$$M(x,y,t) * M(y,z,s) * M(z,u,r) > M(x,u,t+s+r).$$

Since $*$ is minimum t-norm, from the definition of M we have,

$$M(x,y,t) = M(y,z,s) = M(z,u,r) = 1, \quad M(x,u,t+s+r) = 0.$$

And hence,

$$d(x,y) + d(y,z) + d(z,u) < t + s + r \leqslant d(x,u),$$

so this is a contradiction. This completes the proof.

■

Definition 2.9 *Let (X,d) be a metric space and let f be a mapping on X. The mapping f is called Kannan-type mapping if there exists $0 \leqslant \alpha < \frac{1}{2}$ such that,*

$$d(f(x), f(y)) \leqslant \alpha[d(x, f(x)) + d(y, f(y))] \qquad x, y \in X.$$

Example 2.10 Let $f : \mathbb{R} \to \{0, \frac{1}{4}\}$ be defined by

$$f(x) = \begin{cases} \frac{1}{4} & x > 1 \\ 0 & x \leqslant 1 \end{cases}$$

It is easy to check that f is a Kannan-type mapping but it is not continuous.

Definition 2.11 *A function $\psi : [0,1] \times [0,1] \to [0,1]$ is said to be a Ψ-function if*

(i) ψ is monotone increasing and continuous,
(ii) $\psi(x,x) > x$ for all $0 < x < 1$,
(iii) $\psi(1,1) = 1, \psi(0,0) = 0$.

Definition 2.12 *Let $(X, M, *)$ be a generalized fuzzy metric space with minimum t-norm $*$ and let ψ be a Ψ-function. A mapping $f : X \to X$ is called generalized Kannan-type mapping if for all $x, y \in X$*

$$M(f(x), f(y), t) \geqslant \psi(M(x, f(x), \frac{r}{a}), M(y, f(y), \frac{s}{b})), \tag{1}$$

where $r, s > 0$ and $a, b > 0$ with $t = r + s$ and $0 < a + b < 1$.

Definition 2.13 *Let $(X, M, *)$ be a generalized fuzzy metric space. A sequence $\{x_n\} \subseteq X$ is said to converge to $x \in X$ if for all $0 < \varepsilon < 1$ there exists a positive integer N such that for all $n > N$ and $t > 0$, $M(x_n, x, t) > 1 - \varepsilon$.*

Definition 2.14 *Let* $(X, M, *)$ *be a generalized fuzzy metric space. A sequence* $\{x_n\} \subseteq X$ *is said to be a Cauchy sequence if for all* $0 < \varepsilon < 1$ *there exist a positive integer* N *such that* $M(x_n, x_m, t) > 1 - \varepsilon$, *for all* $n, m > N$ *and* $t > 0$.

Definition 2.15 *A generalized fuzzy metric space* $(X, M, *)$ *is said to be complete if every Cauchy sequence in* X *is a convergent sequence.*

3. Fixed Point of Generilized Kannan-Type Mapping

In this section, we prove a fixed point theorem for generalized Kannan-type mappings on a complete generalized fuzzy metric space. Before proving the main theorem a technical lemma should be proved.

Lemma 3.1 *Let* $(X, M, *)$ *be a generalized fuzzy metric space and* f *be a generalized Kannan-type self-map on* X. *Let* $\lim_{t\to\infty} M(x, y, t) = 1$ *for all* $x, y \in X$ *and* $x_n = f(x_{n-1})$ *be an iterative sequence generating by* $x_0 \in X$ *for all* $n \in \mathbb{Z}^+$, *then*

$$\lim_{n\to\infty} M(x_{n+1}, x_n, t) = 1 \qquad t > 0.$$

Proof Let $x_0 \in X$, $x_n = f(x_{n-1})$ be an iterative sequence for all positive integers n and r, s, a and b be positive real numbers with $0 < a+b < 1$. From the inequality (1) of generalized Kannan-type mapping, for $t = r + s$ we have:

$$\begin{aligned} M(x_{n+1}, x_n, t) &= M(f(x_n), f(x_{n-1}), t) \\ &\geqslant \psi(M(x_n, f(x_n), \frac{r}{a}), M(x_{n-1}, f(x_{n-1}), \frac{s}{b})) \\ &= \psi(M(x_n, x_{n+1}, \frac{r}{a}), M(x_{n-1}, x_n, \frac{s}{b})) \\ &= \psi(M(x_{n+1}, x_n, \frac{r}{a}), M(x_n, x_{n-1}, \frac{s}{b})). \end{aligned} \tag{2}$$

For all $t > 0$, putting $r = \frac{at}{a+b}$, $s = \frac{bt}{a+b}$ and $c = a + b$ in (2), we can obtain the following:

$$M(x_{n+1}, x_n, t) \geqslant \psi(M(x_{n+1}, x_n, \frac{t}{c}), M(x_n, x_{n-1}, \frac{t}{c})) \qquad (n \in \mathbb{Z}^+). \tag{3}$$

Now we show that the following inequality holds:

$$M(x_{n+1}, x_n, \frac{t}{c}) \geqslant M(x_n, x_{n-1}, \frac{t}{c}) \qquad t > 0, n \in \mathbb{Z}^+. \tag{4}$$

Proof by contradiction, suppose that there exists $t > 0$ such that $M(x_{n+1}, x_n, \frac{t}{c}) < M(x_n, x_{n-1}, \frac{t}{c})$. By Ψ-function properties (2.11) and the inequality (3), we have

$$\begin{aligned} M(x_{n+1}, x_n, t) &\geqslant \psi(M(x_{n+1}, x_n, \frac{t}{c}), M(x_n, x_{n-1}, \frac{t}{c})) \\ &\geqslant \psi(M(x_{n+1}, x_n, \frac{t}{c}), M(x_{n+1}, x_n, \frac{t}{c})) \\ &> M(x_{n+1}, x_n, \frac{t}{c}) \\ &> M(x_{n+1}, x_n, t), \end{aligned}$$

which is a contradiction. So the inequality (3) and (4) implies that the following desired inequality:

$$M(x_{n+1}, x_n, t) \geqslant M(x_n, x_{n-1}, \frac{t}{c}), \qquad (t > 0, n \in \mathbb{Z}^+).$$

If we apply induction to the above inequality we see that;

$$M(x_{n+1}, x_n, t) \geqslant M(x_1, x_0, \frac{t}{c^n}) \qquad (n \in \mathbb{Z}^+).$$

Our additional assumption on fuzzy metric implies that $\lim_{n\to\infty} M(x_1, x_0, \frac{t}{c^n}) = 1$. Thus, by taking limit as n tends to infinity, it can deduce that

$$\lim M(x_{n+1}, x_n, t) = 1, \qquad t \geqslant 0.$$

■

Now we are ready to state and prove our main results. Let $*$ be the 3-rd order continuous minimum t-norm given by $\alpha * \beta * \gamma = min\{\alpha, \beta, \gamma\}$ and $(X, M, *)$ be a complete generalized fuzzy metric space. We prove the existence fixed point theorem of generalized Kannan-type mapping.

THEOREM 3.2 *Let $(X, M, *)$ be a complete generalized fuzzy metric space, where*

(i) $*$ *is the 3-rd order continuous minimum t-norm,*
(ii) $\lim_{t\to\infty} M(x, y, t) = 1$ *for all* $x, y \in X$,
(iii) $f : X \to X$ *be a generalized Kannan-type mapping.*

Then f *has a unique fixed point.*

Proof Let $x_0 \in X$ and $x_n = f(x_{n-1})$ be an iterative sequence that was constructed in the above lemma, now we show that $\{x_n\}$ is a Cauchy sequence. Suppose that it is not Cauchy, so by definition, there exists $0 < \varepsilon < 1$ for which we can find $t > 0$ and subsequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of $\{x_n\}$ with $n(k) > m(k) > k$ for all positive integers k such that

$$M(x_{m(k)}, x_{n(k)}, t) \leqslant 1 - \varepsilon. \qquad (5)$$

So for all $r, s > 0$ with $t = r + s$ and $a, b > 0$ with $0 < a + b < 1$ we have

$$\begin{aligned} 1-\varepsilon &\geqslant M(x_{m(k)}, x_{n(k)}, t) \\ &= M(f(x_{m(k)-1}), f(x_{n(k)-1}), t) \\ &\geqslant \psi(M(x_{m(k)-1}, f(x_{m(k)-1}), \frac{r}{a}), M(x_{n(k)-1}, f(x_{n(k)-1}), \frac{s}{b})) \\ &\geqslant \psi(M(x_{m(k)-1}, x_{m(k)}, \frac{r}{a}), M(x_{n(k)-1}, x_{n(k)}, \frac{s}{b})). \end{aligned}$$

Therefore,

$$1-\varepsilon \geqslant \psi(M(x_{m(k)-1}, x_{m(k)}, \frac{r}{a}), M(x_{n(k)-1}, x_{n(k)}, \frac{s}{b})). \tag{6}$$

By lemma(3.1), for all $t > 0$ we have

$$\lim_{n\to\infty} M(x_{n+1}, x_n, t) = 1.$$

So we can choose k large enough such that

$$M(x_{m(k)-1}, x_{m(k)}, \frac{r}{a}) > 1-\varepsilon \quad and \quad M(x_{n(k)-1}, x_{n(k)}, \frac{s}{b}) > 1-\varepsilon \tag{7}$$

Therefore, from (6), (7) and the definition of Ψ-function it is inferred that,

$$1-\varepsilon \geqslant \psi(1-\varepsilon, 1-\varepsilon) > 1-\varepsilon \tag{8}$$

which is a contradiction. So, $\{x_n\}$ is a Cauchy sequence and completeness of generalized fuzzy metric space $(X, M, *)$ implies that $\lim_{n\to\infty} x_n = x$ for some $x \in X$. Now we claim that x is a fixed point for f. Assume it is not; so there exists $t > 0$ such that $0 < M(x, f(x), t) < 1$. Since $0 < b < 1$, we can choose $\eta_1, \eta_2, r, s > 0$ such that

$$t = \eta_1 + \eta_2 + r + s \quad and \quad \frac{s}{b} > t. \tag{9}$$

Then we have

$$\begin{aligned} M(x, f(x), t) &\geqslant M(x, x_n, \eta_1) * M(x_n, x_{n+1}, \eta_2) * M(x_{n+1}, f(x), r+s) \\ &\geqslant M(x, x_n, \eta_1) * M(x_n, x_{n+1}, \eta_2) * \psi(M(x_n, f(x_n), \frac{r}{a}), M(x, f(x), \frac{s}{b})) \\ &\geqslant M(x, x_n, \eta_1) * M(x_n, x_{n+1}, \eta_2) * \psi(M(x_n, x_{n+1}, \frac{r}{a}), M(x, f(x), \frac{s}{b})) \end{aligned} \tag{10}$$

By Lemma (3.1) and the convergence of $\{x_n\}$ to x, there exists a positive integer N_1 such that for all $n > N_1$,

$$M(x, x_n, \eta_1) * M(x_n, x_{n+1}, \eta_2) * M(x_n, x_{n+1}, \frac{r}{a}) > M(x, f(x), t).$$

Then from (9) and (10) it follows that,

$$\begin{aligned} M(x,f(x),t) &> M(x,f(x),t) * \psi(M(x,f(x),t), M(x,f(x),\frac{s}{b})) \\ &\geqslant M(x,f(x),t) * \psi(M(x,f(x),t), M(x,f(x),t)) \\ &\geqslant M(x,f(x),t), \end{aligned}$$

which is a contradiction.

Hence $M(x,f(x),t) = 1$ for all $t > 0$, therefore, x is a fixed point for f.
For the uniqueness, suppose that f has two fixed points x and u. Therefore, with all of the above assumptions on a, b and r, s, for all $t > 0$, we have

$$\begin{aligned} M(x,u,t) &= M(f(x), f(u), t) \\ &\geqslant \psi(M(x,f(x),\frac{r}{a}), M(u,f(u),\frac{s}{b})) \\ &= \psi(M(x,x,\frac{r}{a}), M(u,u,\frac{s}{b})) \\ &\psi(1,1) = 1. \end{aligned}$$

Thus $x = u$, which completes the proof. ∎

Finally, an example will help to support our theorem.

Example 3.3 Let $X = \{x_1, x_2, x_3, x_4\}$, $*(\alpha,\beta,\gamma) = \min\{\alpha,\beta,\gamma\}$ and $M(x,y,t)$ be defined as

$$M(x_1,x_2,t) = M(x_2,x_1,t) \begin{cases} 0 & if \quad t \leqslant 0 \\ 0.70 & if \quad 0 < t \leqslant 6 \\ 1 & if \quad t > 6 \end{cases}$$

$$M(x_1,x_3,t) = M(x_3,x_1,t) \begin{cases} 0 & if \quad t \leqslant 0 \\ 0.90 & if \quad 0 < t \leqslant 3 \\ 1 & if \quad t > 3 \end{cases}$$

$$M(x_1,x_4,t) = M(x_4,x_1,t) \begin{cases} 0 & if \quad t \leqslant 0 \\ 0.80 & if \quad 0 < t \leqslant 4 \\ 1 & if \quad t > 4 \end{cases}$$

$$M(x_2,x_3,t) = M(x_3,x_2,t) \begin{cases} 0 & if \quad t \leqslant 0 \\ 0.95 & if \quad 0 < t \leqslant 3 \\ 1 & if \quad t > 3 \end{cases}$$

$$M(x_2, x_4, t) = M(x_4, x_2, t) \begin{cases} 0 & if \quad t \leqslant 0 \\ 0.80 & if \quad 0 < t \leqslant 4 \\ 1 & if \quad t > 4 \end{cases}$$

$$M(x_3, x_4, t) = M(x_4, x_3, t) \begin{cases} 0 & if \quad t \leqslant 0 \\ 0.70 & if \quad 0 < t \leqslant 6 \\ 1 & if \quad t > 6 \end{cases}$$

Then $(X, M, *)$ is a complete generalized fuzzy metric space. Let $f : X \to X$ be given by $f(x_1) = f(x_2) = f(x_3) = x_3$ and $f(x_4) = x_1$. If we take $\psi(x, y) = \frac{\sqrt{x}+\sqrt{y}}{2}$ and $a = 0.2, b = 0.75$, then f satisfies all conditions of the Theorem (3.2) and x_3 is a unique fixed point of f.

4. Conclusion

In this work, we extend Kannan- type mappings to generalized Kannan-type maps in generalized fuzzy metric spaces. Then we proved the existence of unique fixed point theorem for this family of mappings on generalized complete fuzzy metric spaces. We also supported our result by an example.

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