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Inventory Model of Deteriorating Items with Revenue Sharing on Preservation Technology Investment under Price Sensitive Stock Dependent Demand

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Abstract. The objective of this research is to tackle the emerging problem of jointly determining the optimal retail price, the replenishment cycle, and the cost of preservation technology investment from an integrated perspective among the supplier and the manufacturer. For this, we develop an integrated single-manufacturer single-retailer inventory model for deteriorating items under revenue sharing on preservation technology investment. The propose model is develop under the contract that the manufacturer offers the retailer for a percentage of revenue sharing on preservation technology investment. The market demand is assume to be dependent on both the on-hand stock and price, and the manufacturer and the retailer are in an agreement of lot-for-lot policy. Numerical and graphical analysis is given to illustrate the model..

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1. Introduction

To improve the customer service level, increase business competitiveness and reduce the economic losses of the retailer and manufacturer, the effective use of preservation technology become essential for the trade of deteriorating items. Inventory Control play a vital role in supply chain of deteriorating items in order to reduce

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the economic losses. In the last few years it is observed, also recognized by marketing researchers and practitioners, that the demand rate of most of the inventory follows the function of selling price and also influenced by the stock levels of the inventory.

The literature related to this work can be categorize into two parts; first inventory model of deteriorating items with price and stock dependent demand and second the use of preservation technology and cooperative investment.

Inventory of deteriorating items was first studied by Within (1957), he considered the deterioration of fashion goods at the end of prescribed storage period. Ghare and Schrader (1963) extended the classical EOQ formula with exponential decay of inventory due to deterioration and gave a mathematical model of inventory of deteriorating items. Levin et al. (1972) pointed out that large piles of consumer goods displayed in a supermarket would attract the customer to buy more. Baker and Urban (1988) developed an EOQ model for a power function of inventory-level-dependent demand pattern. Padmanabhan and Vrat (1990) formulated a multi-item inventory model of deteriorating items with stock-dependent demand under resource constraints. Datta and Pal (1990) studied an inventory model in which the demand rate is dependent on the instantaneous inventory level until a given inventory level is achieved, after which the demand rate becomes constant. Urban and Baker (1997) studied the EOQ model in which the demand is a multivariate function of price, time, and level of inventory. Wee (1997) formulated a replenishment policy for items with a price-dependent demand and a varying rate of deterioration. Roy and Maiti (1998) developed multi-item inventory models of deteriorating items with stock-dependent demand in a fuzzy environment. In (2001) Goyal and Giri gave recent trends of modeling in deteriorating items inventory. They classified inventory models on the basis of demand variations and various other conditions or constraints. Datta and Paul (2001) analyzed a multi-period EOQ model with stock-dependent and price-sensitive demand rate. Other papers related to this area are Hwang and Hahn (2000), Chang (2004), You (2005), Roy(2008), Tripathy and Mishra (2010), Chang et al. (2010), Hung (2011) and others. Recently Maihami and Kamalabadi (2012) developed a joint pricing and inventory control system for non-instantaneous deteriorating items and adopt a price and time dependent demand function. Chung et al. (2012) gave a complete solution procedures for the mathematical analysis of some families of optimal inventory models with order-size dependent trade credit and deterministic and constant demand.

The deterioration rate of items in the above mentioned papers is viewed as an exogenous variable, which is not subject to control. In practice, the deterioration rate of products can be controlled and reduced through various efforts such as procedural changes and specialized equipment acquisition. The consideration of preservation technology is important due to rapid social changes and need to reduce the economic losses of the retailer in order to increase the competition in the business. The following researchers considered the impact of preservation technology investment, to reduce the deterioration rate in the development of deteriorating inventory model.

Hsu et al. (2010) developed a deteriorating inventory policy when the retailer invests on the preservation technology to reduce the rate of product deterioration. Dye and Hsieh (2012) formulated an optimal replenishment policy for deteriorating items with effective investment in preservation technology. Lee and Dye (2012) formulated a deteriorating inventory model with stock-dependent demand by allowing preservation technology cost as a decision variable in conjunction with replacement policy. Shah and Shah (2013) formulated an optimal pricing policies for deteriorating items with preservation technology and price sensitive demand to determine the optimal value of price, cycle time and preservation technology cost. Mishra (2013a) developed a deteriorating inventory model using preservation technology with salvage value and shortages. Mishra (2013b) extended their model for time dependent demand and holding cost with controllable deterioration rate for instantaneous deteriorating items. Dye (2013) studied the effect of preservation technology investment on a non-instantaneous deteriorating inventory model. Khedlekar (2013) developed a pricing policy with time and price dependent demand for deteriorating items. Yu et al. (2013) developed coordination-based inventory management for deteriorating items in a two-echelon supply chain with profit sharing. In their study they consider that the retailer and the supplier act as the leader and follower and the market demand is affected by the sale price of the product. He and Huang (2013) optimizing inventory and pricing policy for seasonal deteriorating products with preservation technology investment Hsieh and Dye (2013) formulated a production-inventory model for deteriorating items with time-varying demand and finite replenishment rate by allowing preservation technology cost as a decision variable in conjunction with production policy. The objective of their study is to find the optimal production and preservation technology investment strategies while minimizing the total cost over the planning horizon. Mishra (2014) developed an inventory model for non instantaneous deteriorating items with controllable deterioration rate for time-dependent Demand and time-varying Holding cost. Zhang et al. (2014) studied an optimal pricing policy for deteriorating items with preservation technology investment. The goal of their study is to maximize the total profit per unit time by simultaneously determining the optimal selling price, length of replenishment cycle and preservation technology investment. Tayal et al. (2014) developed two echelon supply chain model for deteriorating items with effective investment in preservation technology. In their study they consider that the retailer's demand is seasonal with expiration date and retailer's invests on the preservation technology to reduce the rate of product deterioration. The revenue sharing scheme has studied by Cachon and Lariviere in (2005). They formulated a supply chain coordination scheme with revenue-sharing contracts: strengths and limitations. Giri and Bardhan (2012) formulated the supply chain coordination for a deteriorating item with stock and price-dependent demand under revenue sharing contract. Giovanni and Roselli (2012) demonstrate that shifting from a wholesale price contract to a revenue sharing contract is not payoff-Pareto-improving when the retailer, who is the player transferring the share of revenues, is myopic. Recently Chung et al. (2013) developed an inventory models under conditional trade credit in a supply chain system, Lin and Srivastava (2015) gave a two-warehouse inventory model with quantity discounts and maintenance actions under imperfect production processes and Chung et al. (2015) gave an algorithm for the optimal cycle time and pricing decisions for an integrated inventory system with order-size dependent trade credit in supply chain management.

In most of the cases of the business of deteriorating items the manufacturer and retailer manage their own inventory but the investment in preservation technology equipment is comparatively high in respect of some commodity of business (like agriculture product, Dairy product etc) so manufacturer offer to the retailer for cooperative investment in preservation technology to reduce the deterioration and economic losses. In view of above realistic situation here we develop a supply chain coordination scheme of retailer and manufacturer for deteriorating items with stock and price dependent demand to jointly determining the optimal retail price, the replenishment cycle, and the cost of preservation technology investment from an integrated perspective among the supplier and the manufacturer.

The rest of the paper is organized as follows. The assumptions and notations of the model are introduced in the next section. The mathematical formulation of the model and solution procedure is derived in section 3 and 4 respectively and numerical and graphical analysis is presented in section 5. The article ends with some concluding remarks and scope of future research.

2. Notations and Assumptions

The mathematical model is based on the following notations and assumptions.

2.1 *Notations*

- C Purchase cost per unit.
- p Retail price per unit, $p > C$.
- C_m The setup cost per lot for the manufacturer.
- C_p Manufacturing cost per unit.
- C_r The ordering cost per order of the retailer.
- C_d Deterioration cost per cycle.
- k Cost coefficient of investment.
- h Unit inventory holding cost per unit time.
- ξ Preservation technology (PT) cost for reducing deterioration rate in order to preserve the product, $\xi > 0$.
- θ The deterioration rate.
- $m(\xi)$ Reduced deterioration rate due to use of preservation technology.
- τ_ρ Resultant deterioration rate, $\tau_\rho = (1 - m(\xi))$.
- $D(p, I(t))$ Demand rate.
- q Production rate.
- δ The subsidy proportion given by manufacturer to the retailers for preservation technology investment.
- T The length of cycle time.
- Q The order quantity during a cycle of length T.
- $I_m(t)$ The level of positive inventory at time t for the manufacturer, $0 \leqslant t \leqslant T$.
- $I_r(t)$ The level of positive inventory at time t for the retailer, $0 \leqslant t \leqslant T$.
- TF^R Total profit per unit time for the retailer.
- TF^M Total profit per unit time for the manufacturer.
- TF Total profit per unit time under integrated system.

2.2 *Assumptions*

- The demand rate $D(p, I(t)) = \int_0^T (\alpha + \beta I(t))p^{-\eta} dt; \alpha > 0, \beta > 0, \eta > 1$ is a non-negative power function of selling price and stock level.
- Holding cost is constant.
- Production rate is constant.
- The deterioration rate is constant.
- Preservation technology is used for controlling the deterioration rate.
- Both retailer and manufacturer invest in the preservation technology to reduce the deterioration rate.
- The retailer invests in the preservation technology ξ and manufacturer give subsidy with proportion δ to the retailer's investment.
- The proportion of reduced deterioration rate, $m(\xi)$, is concave increasing function of ξ.

- The holding cost and deterioration cost for both manufacturer and retailer are same.
- The lead time is zero.
- The replenishment rate is infinite.
- The planning horizon is finite.

3. Mathematical Formulation

The rate of change in inventory level for retailer as depicted in fig.-1 occurs due to demand and resultant deterioration rate (τ_ρ) during positive stock period $[0, T]$. Hence, the inventory level for retailer at any time during$[0, T]$ is governed by the following differential equation

$$\frac{dI_r(t)}{dt} + \tau_\rho I_r(t) = -D(p, I_r(t)); 0 \leqslant t \leqslant T. \tag{1}$$

With boundary condition
$I_r(t) = 0$ at $t = T$ and $I_r(t) = Q$ at $t = 0$
While the rate of change in the inventory level of manufacturer as depicted in fig-2, after receiving the retailer order of quantity Q at time tm, occurs due to production and resultant deterioration during $[t_m, T]$. Hence the inventory level for the manufacturer at any time during $[t_m, T]$ is governed by the following differential equation

$$\frac{dI_m(t)}{dt} + \tau_\rho I_m(t) = q; t_m \leqslant t \leqslant T. \tag{2}$$

With boundary condition
$I_m(t_m) = Q$ at $t = 0$ and $I_m(t) = 0$ at $t = T$ftbpFU5.9447in1.785in0ptGraphical

Representation of Inventory System of RetailerFigure-1

ftbpFU6.0796in2.3428in0ptGraphical Representation of Inventory System of manufacturerFigure-2

Now we derive the total profit function for the retailer, for manufacturer and for integrated approach.
Case I: For Retailer
The solution of equation (1) is

$$I_r(t) = \left[\frac{\alpha p^{-\eta}}{\theta(1 - m(\xi)) + \beta p^{-\eta}} \left(e^{(\theta(1-m(\xi))+\beta p^{-\eta})(T-t)} - 1 \right) \right]; 0 \leqslant t \leqslant T. \tag{3}$$

The order size for the retailer during total time interval $[0, T]$ is

$$Q = I_r(0) = \frac{\alpha p^{-\eta}}{\theta(1 - m(\xi)) + \beta p^{-\eta}} \left(e^{(\theta(1-m(\xi))+\beta p^{-\eta})(T)} - 1 \right). \tag{4}$$

The total profit of retailer per replenishment cycle consists of the following components (for convenience $m(\xi)$ is denoted by m):

Inventory holding cost per cycle

$$IHC = h\left(\int_0^T I_r(t)dt\right).$$

$$IHC = \frac{h\alpha(-p^\eta - T\theta p^\eta + T\theta m p^\eta - T\beta + p^\eta e^{(-T(\theta m-\theta-\beta p^{-\eta}))})}{(\theta^2 p^{2\eta} - 2\theta^2 p^{2\eta} m + 2\theta p^\eta \beta + \theta^2 p^{2\eta} m^2 - 2\theta m p^\eta \beta + \beta^2)}. \tag{5}$$

Deterioration cost per cycle

$$DC = C_d\theta(1-m)\int_0^T I_r(t)dt$$

$$DC = \frac{C_d\theta(1-m)\alpha(-p^\eta - T\theta p^\eta + T\theta m p^\eta - T\beta + p^\eta e^{(-T(\theta m-\theta-\beta p^{-\eta}))})}{(\theta^2 p^{2\eta} - 2\theta^2 p^{2\eta} m + 2\theta p^\eta \beta + \theta^2 p^{2\eta} m^2 - 2\theta m p^\eta \beta + \beta^2)}. \tag{6}$$

Preservation technology investment cost

$$PRC = (1-\delta)k\xi T. \tag{7}$$

Sale revenue

$$SR = pTD(p, I_r(t))$$

$$SR = \frac{pT\alpha(p^\eta T\theta^2 - 2p^\eta T\theta^2 m + T\theta\beta + p^\eta T\theta^2 m^2 - T\theta m\beta - \beta + \beta e^{(-T(\theta m-\theta-\beta p^{-\eta}))})}{(\theta^2 p^{2\eta} - 2\theta^2 p^{2\eta} m + 2\theta p^\eta \beta + \theta^2 p^{2\eta} m^2 - 2\theta m p^\eta \beta + \beta^2)}. \tag{8}$$

Purchase cost

$$PC = C * Q.$$

$$PC = \frac{C\alpha p^{-\eta}\left(e^{(\theta(1-m)+\beta p^{-\eta})(T)} - 1\right)}{\theta(1-m) + \beta p^{-\eta}}. \tag{9}$$

Ordering cost

$$OC = C_r. \tag{10}$$

Thus the total profit function for retailer per time unit is given by,

$TF^R = \frac{1}{T}$[Sales revenue-Purchase cost-Ordering cost-Holding cost-Deterioration cost

-Preservation technology investment cost]

$$TF^R = \left[\begin{array}{c} \frac{\left(\begin{array}{c} pT\alpha(p^\eta T\theta^2 - 2p^\eta T\theta^2 m + T\theta\beta + p^\eta T\theta^2 m^2 - T\theta m\beta - \beta \\ +\beta e^{(-T(\theta m - \theta - \beta p^{-\eta}))} \end{array} \right) - (\alpha(-p^\eta - T\theta p^\eta + T\theta m p^\eta - T\beta + p^\eta e^{(-T(\theta m - \theta - \beta p^{-\eta}))})(h + C_d\theta(1-m)))}{(\theta^2 p^{2\eta} - 2\theta^2 p^{2\eta} m + 2\theta p^\eta \beta + \theta^2 p^{2\eta} m^2 - 2\theta m p^\eta \beta + \beta^2)} \\ -\frac{C\alpha p^{-\eta}\left(e^{(\theta(1-m)+\beta p^{-\eta})(T)} - 1\right)}{\theta(1-m)+\beta p^{-\eta}} - (1-\delta)k\xi T - C_r. \end{array} \right] \tag{11}$$

Case II: For Manufacturer

The solution of equation (2) is

$$I_m = \left[\frac{q}{\theta(1-m(\xi))}(1 - e^{\theta(1-m(\xi))(t_m - t)}) \right]. \tag{12}$$

The starting time of production for the manufacturer during time interval [0, T] is

$$t_m = T + \tfrac{1}{\theta(1-m(\xi))} \ln\left(1 - \tfrac{Q}{q}\theta(1-m(\xi))\right)$$

From equation (4)

$$t_m = T + \frac{1}{\theta(1-m(\xi))} \ln\left(1 - \frac{\left[\frac{\alpha p^{-\eta}}{\theta(1-m(\xi))+\beta p^{-\eta}} \left(e^{(\theta(1-m(\xi))+\beta p^{-\eta})(T)} - 1\right)\right]\theta(1-m(\xi))}{q}\right). \tag{13}$$

The total profit of manufacturer per replenishment cycle consists of the following components (for convenience $m(\xi)$is denoted by m):

Inventory holding cost per cycle;

$$IHC = h\left(\int_{t_m}^{T} I_m(t)dt\right)$$

$$IHC = \frac{-hq(1 - \theta m t_m + \theta t_m + T\theta m - T\theta - e^{\theta(m-1)(T-t_m)})}{\theta^2(m-1)^2}. \tag{14}$$

Deterioration Cost per cycle

$$DC = C_d\theta(1-m)\int_{t_m}^{T} I_m(t)dt$$

$$DC = \frac{-C_d\theta(1-m)q(1 - \theta m t_m + \theta t_m + T\theta m - T\theta - e^{\theta(m-1)(T-t_m)})}{\theta^2(m-1)^2}. \tag{15}$$

Preservation technology investment

$$PRC = \delta k\xi T. \tag{16}$$

Sale revenue

$$SR = CQ$$

$$SR = \frac{C\alpha p^{-\eta}\left(e^{(\theta(1-m+\beta p^{-\eta})(T)} - 1\right)}{\theta(1-m)+\beta p^{-\eta}}. \tag{17}$$

Manufacturing cost

$$MC = C_p q(T - t_m). \tag{18}$$

Ordering Cost

$$OC = C_m. \tag{19}$$

Thus the total profit function for manufacturer per time unit is given by,
$TF^M = \frac{1}{T}$[Sales Revenue - Manufacturing cost - Ordering cost - Holding cost – Deterioration cost-Preservation technology investment cost]

$$TF^M = \frac{1}{T}\left[\begin{array}{c}(-h - C_d\theta(1-m))\left(\begin{array}{c}\frac{C\alpha p^{-\eta}\left(e^{(\theta(1-m+\beta p^{-\eta})(T)}-1\right)}{\theta(1-m)+\beta p^{-\eta}} \\ -\frac{q(1-\theta m t_m+\theta t_m+T\theta m-T\theta-e^{\theta(m-1)(T-t_m)})}{\theta^2(m-1)^2}\end{array}\right) \\ -C_p q(T-t_m) - \delta k\xi T - C_m\end{array}\right]. \tag{20}$$

Therefore the total profit per replenishment cycle for whole inventory system is:

$$TF = TF^R + TF^M$$

$$TF = \frac{1}{T}\left[\begin{array}{c}\dfrac{\left(\begin{array}{c}pT\alpha(p^\eta T\theta^2 - 2p^\eta T\theta^2 m + T\theta\beta + p^\eta T\theta^2 m^2 - T\theta m\beta - \beta \\ +\beta e^{(-T(\theta m-\theta-\beta p^{-\eta}))})\end{array}\right) \\ -(\alpha(-p^\eta - T\theta p^\eta + T\theta m p^\eta - T\beta + p^\eta e^{(-T(\theta m-\theta-\beta p^{-\eta}))})(h + C_d\theta(1-m)))}{(\theta^2 p^{2\eta} - 2\theta^2 p^{2\eta} m + 2\theta p^\eta\beta + \theta^2 p^{2\eta} m^2 - 2\theta m p^\eta\beta + \beta^2)} \\ -(-h - C_d\theta(1-m))\left(\frac{q(1-\theta m t_m+\theta t_m+T\theta m-T\theta-e^{\theta(m-1)(T-t_m)})}{\theta^2(m-1)^2}\right) \\ -C_p q(T-t_m) - k\xi T - C_m - C_r.\end{array}\right]. \tag{21}$$

4. Solution Procedure

We consider that the manufacturer and the retailer both are integrated together as a one unit and manufacturer and the retailer both invested for preservation technology and manufacturer give subsidy δ for a portion of investment on preservation technology by retailer. The objective of this integrated system is to find the optimal values of retail price (p), preservation technology investment (ξ) and replenishment cycle time (T) in order to maximize the profit.

The condition for finding the optimal value of p, T and ξ for maximizing the total profit function is and $TF(p, T, \xi)$ is $\frac{\partial TF}{\partial p} = 0$, $\frac{\partial TF}{\partial T} = 0$ and $\frac{\partial TF}{\partial \xi} = 0$ provided the determinant of principal minor of hessian matrix (H-matrix) of $TF(p, T, \xi)$ is negative semi definite. i.e. $det(H_1) \leqslant 0, det(H_2) \geqslant 0, det(H_3) \leqslant 0$ where H_1, H_2, H_3 is the principal minor of the H-matrix.

The H-matrix of function $TF(p, T, \xi)$ is defined as

$$H = \begin{bmatrix} \frac{\partial^2 TF}{\partial p^2} & \frac{\partial^2 TF}{\partial p \partial T} & \frac{\partial^2 TF}{\partial p \partial \xi} \\ \frac{\partial^2 TF}{\partial T \partial p} & \frac{\partial^2 TF}{\partial T^2} & \frac{\partial^2 TF}{\partial T \partial \xi} \\ \frac{\partial^2 TF}{\partial \xi \partial p} & \frac{\partial^2 TF}{\partial \xi \partial T} & \frac{\partial^2 TF}{\partial \xi^2} \end{bmatrix}$$

For this we present the following algorithm to obtain the optimum values of (P, T, ξ)

Algorithm for solution of the model

Step-1: Start

Step-2: Initialize the value of the variable $\alpha, \beta, q, \theta, \delta, \eta, h, C_d, C_p, C_r, C_m,$ and $m(\xi)$.

Step-3: Evaluate $TF(p, T, \xi)$.

Step-4: Evaluate $\frac{\partial TF}{\partial p}, \frac{\partial TF}{\partial T}$ and $\frac{\partial TF}{\partial \xi}$.

Step-5: Solve the simultaneous equation and $\frac{\partial TF}{\partial p} = 0, \frac{\partial TF}{\partial T} = 0$ and $\frac{\partial TF}{\partial \xi} = 0$.

Step-6: Choose one set of solution from step-4.

Step-7: Evaluate $PM1 = det(H1), PM2 = det(H2)$ and $PM3 = det(H3)$ (where H1, H2, H3 is the principal minor of the H-matrix).

Step-8: If the value of $PM1 \leqslant 0, PM2 \leqslant 0$ and $PM3 \leqslant 0$ then this set of solution is optimal and go to step 10.

Step-9: otherwise go to step-5

Step-10: End

Since the nature of profit function is highly non-linear so the condition of optimality is to be verifying numerically and graphically in the next section by using the maple mathematical software according to the above defined algorithm.

5. Numerical and Graphical Illustration

For numerical and graphical illustration we consider an inventory system with the following parameter in proper unit $\alpha = 100, \beta = 2, q = 40, \theta = 0.2, k = 3, \eta = 2, h = 2, C_p = 3, C_d = 2, C_r = 100, C_m = 200$ and $m(\xi) = (1 - e^{-0.5\xi})$. By using maple mathematical software the computer output of the program of optimal values of retail price (p), preservation technology investment (ξ) and replenishment cycle time (T) is $T = 5.2300, p = 3.2928, \xi = 9.5989$and the value of total profit function is $TF = 28.8518$.

The 3-D graph of total profit function with values of retail price (p) varies from 3.00 to 4.00 and preservation technology investment (ξ) with equal interval from 9.0 to 10.8 at fixed optimal replenishment cycle time T at 5.23 is strictly concave graph of total cost function (TF) as depicted in the figure 3.

ftbpFU4.4252in2.4933in0ptTotal profit (TF) vs. Retail price (p) and Preservation technology investment (ξ).Figure 3

Also the 2-D graph of total profit function with values of retail price(p) varies from 3.00 to 4.00 with equal interval at fixed optimal preservation technology investment (ξ) at 9.59 and replenishment cycle time (T) at 5.23, total profit function with values of preservation technology investment (ξ) varies from 9.0 to 10.0 with equal interval at fixed optimal retail price (p) at 3.29 and fixed replenishment cycle time (T) at 5.23 is strictly concave graph of total cost function (TF), as depicted in the figure 4 and 5 respectively.

ftbpFU4.8542in2.1923in0ptTotal Profit (TF) vs. Retail price (p) at fixed Preservation technology investment (ξ).Figure 4ftbpFU5.2338in2.2001in0ptTotal Profit

(TF) vs. Preservation technology investment (ξ) at fixed Retail price (p).Figure 5

The above numerical and graphical analysis indicate that the total profit function is strictly concave function and thus the optimal values of retail price (p), preservation technology investment (ξ) and replenishment cycle time (T) can be determined uniquely with maximization of total profit function.

6. Conclusions

The purpose of this study is to develop a coordination scheme for inventory of deteriorating items with revenue sharing by manufacturer on preservation technology investment of retailer under price sensitive and stock dependent demand. The products with high deterioration rate are always crucible to the retailer's business. In real markets, the retailer can reduce the deterioration rate of product by making effective capital investment in warehouse equipment but some retailer cannot afford the investment on preservation technology equipment. In this study to reduce the deterioration rate retailer invested in the preservation technology and manufacturer give subsidy on a portion of preservation technology investment and a solution procedure has developed to determine an optimal values of retail price, preservation technology investment and replenishment cycle time with maximization of total profit of manufacturer and retailer. A numerical and graphical illustration has presented to illustrate the model and check the condition of optimality and stability of the solution. Our recommendation to such manufacturer and retailer who follow the integrated approach: they can optimize their profit by deciding the optimal values of retail price, preservation technology investment and replenishment cycle of inventory in their warehouses using this model. This model can further be extended by taking more realistic assumptions such as time dependent deterioration, time varying holding cost, probabilistic demand rate, allow the shortages etc.

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