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Haar Wavelet and Adomain Decomposition Method for Third Order Partial Differential Equations Arising in Impulsive Motion of a Flat Plate

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Abstract. We present here, a Haar wavelet method for a class of third order partial differential equations (PDEs) arising in impulsive motion of a flat plate. we also, present Adomain decomposition method to find the analytic solution of such equations. Efficiency and accuracy have been illustrated by solving numerical examples.

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1. Introduction

Many real life problems are formulate in the form of mathematical models of ordinary and partial differential equations (PDEs). Third order PDEs are arising in many areas of physics, mathematics and engineering. Impulsive motion of a flat-plat Korteweg-de Vries layer equation with pressure and hydrodynamic boundary layer equations are formulated mathematically in the form of partial differential equations of order third. The general third order PDEs is of the form:

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$$u_{ttt} = f(x, t, u, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, \frac{\partial^3 u}{\partial x^3}, ..., \frac{\partial^2 u}{\partial t^2}, \frac{\partial^3 u}{\partial t^3}), \tag{1}$$

with initial and boundary conditions are given below as:

$$u(x, 0) = f_1(x), \frac{\partial u}{\partial t}(x, 0) = f_2(x), \frac{\partial^2 u}{\partial t^2}(x, 0) = f_3(x), u(0, t) = g_1(t), u(1, t) = g_2(t). \tag{2}$$

We discuss here, the solutions of PDEs of the form:

$$\gamma \frac{\partial^3 u}{\partial t^3}(x, t) = \beta \frac{\partial^2 u}{\partial x^2}(x, t) - \eta u(x, t) + q(x, t), \tag{3}$$

where γ, β and η are constants and f_1, f_2, f_3, g_1 and g_2 are known functions. Equations of the form (3) with initial and boundary conditions have been studied in [12, 15, 16]. Direct numerical methods have been discussed in [12] for obtaining the solution of a class of third-order partial differential equations. In [15], numerical solutions of third-order partial differential equations arising in the impulsive motion of a flat plate is presented. An analytic study on the third-order dispersive partial differential equations has been carried out in [16]. The impulsive motion of a flat plate in a viscoelastic fluid has been discussed in [13]. In [4], Haar wavelet method for solving lumped and distributed- parameter systems has been presented. Numerical solution of evolution equations by the Haar wavelet method has been presented in [11]. Numerical solutions of Fisher's equation, FitzHugh-Nagumo equation and generalized Burger-Huxley equation with Haar wavelet methods have been presented in [3, 6, 7] respectively. In [17], analytic study on Burgers, Fisher, Huxley equations and combined forms of these equations have been presented. Numerical solution of differential equations have been presented in [10] using Haar wavelet. Application of the Haar wavelet transform has been used for solving integral and differential equations in [9]. Higher order ordinary and partial differential equations have been solved in [2, 8, 14] by using different numerical techniques.

In Section 2, we briefly describe Adomain decomposition method. In Section 3, Haar wavelet method is described. Function approximation is presented in Section 4. In Section 5, methods for solving third-order partial differential equations have been presented, and in Section 6, numerical examples have been solved using the present method to illustrate the efficiency and accuracy of present method.

2. Adomain Decomposition Method

Consider a third order non-homogeneous partial differential equation of the form

$$\gamma \frac{\partial^3 u}{\partial t^3}(x, t) = \beta \frac{\partial^2 u}{\partial x^2}(x, t) - \eta u(x, t) + q(x, t), \tag{4}$$

with initial and boundary conditions. Rewrite it in the operator form as:

$$\gamma L_t(u) = \beta L_x(u) - \eta u(x,t) + q(x,t), \tag{5}$$

where the differential operator are defined as:

$$L_t(.) = \frac{\partial^3}{\partial t^3}(.), \qquad L_x(.) = \frac{\partial^2}{\partial x^2}(.). \tag{6}$$

The inverse operator is defined as:

$$L_t^{-1}(.) = \int_0^t \int_0^t \int_0^t (.)dtdtdt, \qquad L_x^{-1}(.) = \int_0^x \int_0^x (.)dxdx. \tag{7}$$

Applying the inverse operator on both side of (5), we obtain:

$$L_t^{-1}\gamma L_t(u) = L_t^{-1}(\beta L_x(u) + \varphi(u) + q(x,t)), \tag{8}$$

where $\varphi(u) = -\eta u(x,t)$. Applying integration on (8), we obtain:

$$\gamma u(x,t) = \gamma u(x,0) + \gamma t u_t(x,0) + \gamma\frac{t^2}{2}u_{tt}(x,0) + L_t^{-1}(\beta L_x(u) + \varphi(u) + q(x,t)). \tag{9}$$

Using the concept discussed in [1], we write the solution $u(x,t)$ in the series form as:

$$u(x,t) = \sum_{k=0}^{\infty} u_k(x,t). \tag{10}$$

For solutions in the t directions, use the following recursive relations:

$$u_0 = \gamma u(x,0) + \gamma t u_t(x,0) + \gamma\frac{t^2}{2}u_{tt}(x,0) + L_t^{-1}q(x,t), \tag{11}$$

and

$$u_{k+1}(x,t) = L_t^{-1}(\beta L_x(u_k) + B_k), \qquad k \geqslant 0, \tag{12}$$

where the Adomain polynomials B_n are:

$$B_k = \frac{1}{k!}\frac{d^k}{d\lambda^k}[\varphi(\sum_{k=0}^{\infty}\lambda^k u_k)], \tag{13}$$

From here, components $u_0, u_1, u_2, ...$ are calculated. Finally, the series solution is obtained from (10).

3. Haar Wavelets

The Haar functions are an orthogonal family of switched rectangular waveforms where amplitudes can differ from one function to another. They are defined in the interval $[0, 1]$ as below:

$$H_I(x) = \begin{cases} 1, & \alpha_1 \leqslant x < \alpha_2, \\ -1, & \alpha_2 \leqslant x < \alpha_3, \\ 0, & \text{elsewhere}, \end{cases} \tag{14}$$

where $\alpha_1 = \frac{K}{N}$, $\alpha_2 = \frac{K+0.5}{N}$ and $\alpha_3 = \frac{K+1}{N}$. Integer $N = 2^R$, $(R = 0, 1, 2, 3, 4, ...J)$ indicates the level of the wavelet, and $K = 0, 1, 2, 3, ..., N-1$ is the translation parameter. J represents the maximal level of resolution. The index I is calculated according the formula $I = N+K+1$. In the case of minimal values, $N = 1$, $K = 0$ we have $I = 2$. The maximal value of I is $I = 2M$, where $M = 2^J$. It is assumed that for value $I = 1$, corresponding scaling function in $[0, 1]$ is:

$$H_1(x) = \begin{cases} 1, & 0 \leqslant x \leqslant 1, \\ 0, & \text{elsewhere}. \end{cases} \tag{15}$$

The operational matrices of integration, which are $2M \times 2M$ square matrices, are obtained from the following relations:

$$P_{1,I}(x) = \begin{cases} x - \alpha_1, & x \in [\alpha_1, \alpha_2), \\ \alpha_3 - x, & x \in [\alpha_2, \alpha_3), \\ 0, & \text{elsewhere}; \end{cases} \tag{16}$$

$$P_{2,I}(x) = \begin{cases} \frac{1}{2}(x-\alpha_1)^2, & x \in [\alpha_1, \alpha_2), \\ \frac{1}{4m^2} - \frac{1}{2}(\alpha_3 - x)^2, & x \in [\alpha_2, \alpha_3), \\ \frac{1}{4m^2}, & x \in [\alpha_3, 1), \\ 0, & \text{elsewhere}; \end{cases} \tag{17}$$

$$P_{3,I}(x) = \begin{cases} \frac{1}{6}(x-\alpha_1)^3, & x \in [\alpha_1, \alpha_2), \\ \frac{1}{4m^2}(x-\alpha_2) - \frac{1}{6}(\alpha_3 - x)^3, & x \in [\alpha_2, \alpha_3), \\ \frac{1}{4m^2}(x-\alpha_2), & x \in [\alpha_3, 1), \\ 0, & \text{elsewhere}; \end{cases} \tag{18}$$

$$P_{4,I}(x) = \begin{cases} \frac{1}{24}(x-\alpha_1)^4, & x \in [\alpha_1, \alpha_2), \\ \frac{1}{8m^2}(x-\alpha_2)^2 - \frac{1}{24}(\alpha_3 - x)^4 + \frac{1}{192m^4}, & x \in [\alpha_2, \alpha_3), \\ \frac{1}{8m^2}(x-\alpha_2)^2 + \frac{1}{192m^4}, & x \in [\alpha_3, 1), \\ 0, & \text{elsewhere}. \end{cases} \tag{19}$$

Let us define the collocation points $x_L = \frac{(L-0.5)}{2M}$, where $L = 1, 2, 3, ..., 2M$ and to obtain the operational matrices, discretize the functions $H_I(x)$, $P_{1,I}(x)$, $P_{2,I}(x)$, $P_{3,I}(x)$ and $P_{4,I}(x)$ respectively.

4. Function Approximation

We know that all the Haar wavelets are orthogonal to each other as:

$$\int_0^1 H_I(x)H_l(x)dx = \begin{cases} 2^{-R}, I = L = 2^R + K, \\ 0, I \neq L. \end{cases} \tag{20}$$

Therefore, they construct a very good transform basis. Any square integrable function $y(x)$ in the interval $[0, 1]$ can be expanded by a Haar series of infinite terms:

$$y(x) = c_0H_0(x) + \sum_{R=0}^{\infty}\sum_{K=0}^{2^R-1} c_{2^R+K}H_{2^R+K}(x), \qquad x \in [0, 1] \tag{21}$$

where the Haar coefficients c_I are determined as:

$$c_0 = \int_0^1 y(x)H_0(x)dx, \tag{22}$$

$$c_I = 2^R\int_0^1 y(x)H_I(x)dx, \tag{23}$$

where $I = 2^R + K$, $R\geqslant 0$ and $0\leqslant K < 2^R$, $x\in[0, 1]$ such that the following integral square error ε is minimized:

$$\varepsilon = \int_0^1 [y(x) - \sum_{I=1}^{N} c_IH_I(x)]^2dx, \tag{24}$$

where $N = 2^R$ and $R = 0, 1, 2, 3, ...$ Also, from (21), we write:

y(x)= ${c_N}^TH_N(x)$ =$y_N(x)$, $x \in [0, 1]$,

where the coefficients ${c_N}^T$ and the Haar function vectors $H_N(x)$ are defined as:
${c_N}^T = [c_1, c_2, c_3, ..., c_N]$ and $H_N(x) = [H_1(x), H_2(x), H_3(x), ..., H_N(x)]^T$,
where T represents the transpose of a matrix.

5. Method for Solving Third-Order Partial Differential Equations

Suppose the solution is approximated as:

$$\dddot{u}''(x, t) = \sum_{I=1}^{2M} a_IH_I(x). \tag{25}$$

Integrating above equation, thrice with respect to t, from t_s to t, we successively obtain:

$$\ddot{u}''(x, t) = \ddot{u}''(x, t_s) + (t - t_s)\sum_{I=1}^{2M} a_IH_I(x), \tag{26}$$

$$\dot{u}''(x,t) = \dot{u}''(x,t_s) + (t-t_s)\ddot{u}''(x,t_s) + \frac{(t-t_s)^2}{2}\sum_{I=1}^{2M} a_I H_I(x), \tag{27}$$

$$u''(x,t) = u''(x,t_s) + (t-t_s)\dot{u}''(x,t_s) + \frac{(t-t_s)^2}{2}\ddot{u}''(x,t_s) + \frac{(t-t_s)^3}{6}\sum_{I=1}^{2M} a_I H_I(x). \tag{28}$$

Again, integrating (25), twice with respect to x from 0 to x, we successively obtain:

$$\dddot{u}'(x,t) = \dddot{u}'(0,t) + \sum_{I=1}^{2M} a_I P_{1,I}(x), \tag{29}$$

$$\dddot{u}(x,t) = \dddot{u}(0,t) + x\dddot{u}'(0,t) + \sum_{I=1}^{2M} a_I P_{2,I}(x). \tag{30}$$

Again, integrating (30) thrice with respect to t, from t_s to t, we successively obtain:

$$\ddot{u}(x,t) = \ddot{u}(x,t_s) + (\ddot{u}(0,t) - \ddot{u}(0,t_s)) + x(\ddot{u}'(0,t) - \ddot{u}'(0,t_s)) + (t-t_s)\sum_{I=1}^{2M} a_I P_{2,I}(x), \tag{31}$$

$$\begin{aligned}\dot{u}(x,t) &= \dot{u}(x,t_s) + (t-t_s)\ddot{u}(x,t_s) + (\dot{u}(0,t) - \dot{u}(0,t_s) - (t-t_s)\ddot{u}(0,t_s)) \\ &\quad + x(\dot{u}'(0,t) - \dot{u}'(0,t_s) - (t-t_s)\ddot{u}'(0,t_s)) + \frac{(t-t_s)^2}{2}\sum_{I=1}^{2M} a_I P_{2,I}(x),\end{aligned} \tag{32}$$

$$\begin{aligned}u(x,t) &= u(x,t_s) + (t-t_s)\dot{u}(x,t_s) + \frac{(t-t_s)^2}{2}\ddot{u}(x,t_s) \\ &\quad + (u(0,t) - u(0,t_s) - (t-t_s)\dot{u}(0,t_s) - \frac{(t-t_s)^2}{2}\ddot{u}(0,t_s)) \\ &\quad + x(u'(0,t) - u'(0,t_s) - (t-t_s)\dot{u}'(0,t_s) - \frac{(t-t_s)^2}{2}\ddot{u}'(0,t_s)) + \frac{(t-t_s)^3}{6}\sum_{I=1}^{2M} a_I P_{2,I}(x).\end{aligned} \tag{33}$$

Putting $x = 1$, in (30)-(33), and applying initial and boundary conditions, we successively obtain:

$$\dddot{u}'(0,t) = \dddot{g}_2(t) - \dddot{g}_1(t) - \sum_{I=1}^{2M} a_I P_{2,I}(1), \tag{34}$$

$$\ddot{u}'(0,t) - \ddot{u}'(0,t_s) = \ddot{g}_2(t) - \ddot{g}_2(t_s) - \ddot{g}_1(t) + \ddot{g}_1(t_s) - (t-t_s)\sum_{I=1}^{2M} a_I P_{2,I}(1), \quad (35)$$

$$\dot{u}'(0,t) - \dot{u}'(0,t_s) - (t-t_s)\ddot{u}'(0,t_s) = \dot{g}_2(t) - \dot{g}_2(t_s) - (t-t_s)\ddot{g}_2(t_s)$$
$$- \dot{g}_1(t) + \dot{g}_1(t_s) + (t-t_s)\ddot{g}_1(t_s) - \frac{(t-t_s)^2}{2}\sum_{I=1}^{2M} a_I P_{2,I}(1), \quad (36)$$

$$u'(0,t) - u'(0,t_s) - (t-t_s)\dot{u}'(0,t_s) - \frac{(t-t_s)^2}{2}\ddot{u}''(0,t_s) = g_2(t) - g_2(t_s)$$
$$-(t-t_s)\dot{g}_2(t_s) - \frac{(t-t_s)^2}{2}\ddot{g}_2(t_s) - (g_1(t) - g_1(t_s) - (t-t_s)\dot{g}_1(t_s) - \frac{(t-t_s)^2}{2})$$
$$- \frac{(t-t_s)^3}{6}\sum_{I=1}^{2M} a_I P_{2,I}(1), \quad (37)$$

Substituting the value from (34)-(37) into (29) to (33), and discretizing as $x \to x_L$ and $t \to t_{s+1}$, and let $\triangle t = t_{s+1} - t_s$ on successive integrations, we obtain:

$$\ddot{u}''(x_L, t_{s+1}) = \ddot{u}''(x_L, t_s) + \triangle t\sum_{I=1}^{2M} a_I h_I(x_L), \quad (38)$$

$$\dot{u}''(x_L, t_{s+1}) = \dot{u}''(x_L, t_s) + \triangle t\ddot{u}''(x_L, t_s) + \frac{(\triangle t)^2}{2}\sum_{I=1}^{2M} a_I h_I(x_L), \quad (39)$$

$$u''(x_L, t_{s+1}) = u''(x_L, t_s) + \triangle t\dot{u}''(x_L, t_s) + \frac{(\triangle t)^2}{2}\ddot{u}''(x_L, t_s) + \frac{(\triangle t)^3}{6}\sum_{I=1}^{2M} a_I h_I(x_L), \quad (40)$$

$$\dddot{u}'(x_L, t_{s+1}) = \dddot{g}_2(t_{s+1}) - \dddot{g}_1(t_{s+1}) + \sum_{I=1}^{2M} a_I(P_{1,I}(x_L) - P_{2,I}(1)), \quad (41)$$

$$\dddot{u}((x_L, t_{s+1})) = \dddot{g}_1(t_{s+1}) + x_L(\dddot{g}_2(t_{s+1}) - \dddot{g}_1(t_{s+1})) + \sum_{I=1}^{2M}(a_I(P_{2,I}(x_L) - x_L P_{2,I}(1))), \quad (42)$$

$$\ddot{u}(x_L, t_{s+1}) = \ddot{u}(x_L, t_s) + (\ddot{g}_1(t_{s+1}) - \ddot{g}_1(t_s)) + x_L(\ddot{g}_2(t_{s+1}) - \ddot{g}_2(t_s) - \ddot{g}_1(t_{s+1}) + \ddot{g}_1(t_s))$$
$$+ (\triangle t)\sum_{I=1}^{2M}(a_I(P_{2,I}(x_L) - x_L P_{2,I}(1))), \quad (43)$$

$$\begin{aligned}\dot{u}(x_L,t_{s+1}) &= \dot{u}(x_L,t_s)+\triangle t\ddot{u}(x_L,t_s)+(\dot{g}_1(t_{s+1})-\dot{g}_1(t_s)-\triangle t\ddot{g}_1(t_s))\\ &\quad + x_L(\dot{g}_2(t_{s+1})-\dot{g}_2(t_s)-\triangle t\ddot{g}_2(t_s)-\dot{g}_1(t_{s+1})+\dot{g}_1(t_s)+\triangle t\ddot{g}_1(t_s))\\ &\quad + \frac{(\triangle t)^2}{2}\sum_{I=1}^{2M}(a_I(P_{2,I}(x_L)-x_LP_{2,I}(1))), \end{aligned} \tag{44}$$

$$\begin{aligned}u(x_L,t_{s+1}) &= u(x_L,t_s)+\triangle t\dot{u}(x_L,t_s)+\frac{(\triangle t)^2}{2}\ddot{u}(x_L,t_s)\\ &\quad +(1-x_L)(g_1(t_{s+1})-g_1(t_s)-\triangle t\dot{g}_1(t_s)-\frac{(\triangle t)^2}{2}\ddot{g}_1(t_s))\\ &\quad + x_L(g_2(t_{s+1})-g_2(t_s)-\triangle t\dot{g}_2(t_s)-\frac{(\triangle t)^2}{2}\ddot{g}_2(t_s))\\ &\quad + \frac{(\triangle t)^3}{6}\sum_{I=1}^{2M}(a_I(P_{2,I}(x_L)-x_LP_{2,I}(1))), \end{aligned} \tag{45}$$

In the given scheme,

$$\gamma\frac{\partial^3 u}{\partial t^3}(x_L,t_{s+1})=\beta\frac{\partial^2 u}{\partial x^2}(x_L,t_{s+1})-\eta u(x_L,t_{s+1})+q(x_L,t_{s+1}), \tag{46}$$

from here, wavelet coefficients are calculated and numerical solutions are obtained from (45). It is obtained from (17), that:

$$P_{2,I}(1)=\begin{cases}\frac{1}{2}, & I=1,\\ \frac{1}{4m^2}, & I>1,\end{cases} \tag{47}$$

6. Numerical Examples

Here, we take few examples of third order homogeneous and non-homogeneous partial differential equations to show the efficiency and accuracy of Haar wavelet method. we use MATLAB package to perform all computational work.

Example 1: (Homogeneous) Consider the following third order partial differential equation

$$\frac{\partial^3 u}{\partial t^3}=-4(\frac{\partial^2 u}{\partial x^2}-u), \tag{48}$$

with initial conditions:

$$u(x,0)=\cos x;\ \frac{\partial u}{\partial t}(x,0)=-2\cos x;\ \frac{\partial^2 u}{\partial t^2}(x,0)=4\cos x.$$

and boundary conditions:

$$u(0,t)=e^{-2t};\ u(1,t)=e^{-2t}\cos 1.$$

In the operator form, (48) is written as:

$$L_t(u) = L_x(4u_{xx} - 4u), \tag{49}$$

Applying inverse operator on both side of above equation and simplifying the equation, we obtain:

$$u(x,t) = u(x,0) + tu_t(x,0) + \frac{t^2}{2}u_{tt}(x,0) + L_t^{-1}(L_x(4u_{xx} - 4u)). \tag{50}$$

For solving in the t directions, using the following relations:

$$u_0 = u(x,0) + tu_t(x,0) + \frac{t^2}{2}u_{tt}(x,0), \tag{51}$$

and

$$u_{k+1}(x,t) = L_t^{-1}(L_x(4u_{xx} - 4u)), \quad k \geqslant 0. \tag{52}$$

From here, components $u_0, u_1, u_2, ...$ are calculated as:

$$u_0 = \cos x(1 - 2t + 2t^2), \tag{53}$$

$$u_1 = \cos x(-\frac{4}{3}t^3 + \frac{2}{3}t^4 - \frac{4}{15}t^5), \tag{54}$$

$$u_2 = \cos x(\frac{4}{45}t^6 - \frac{8}{315}t^7 + \frac{2}{315}t^8), \tag{55}$$

and so on. Finally, the series solution is obtained from (10). The solution obtained by Adomain decomposition method is:

$$u(x,t) = e^{-2t}\cos x. \tag{56}$$

The comparison of exact and numerical solutions by using Haar wavelet method for Example 1, is presented in Table 1 for $t = 0.01$ and $J = 3$ with $\Delta t = 0.00001$. Comparison of numerical solutions obtained by Haar wavelet method and Adomain decomposition method at different t are shown in Figure 1.

Example 2: (Non-homogeneous) Consider the following third order partial differential equation

$$\frac{\partial^3 u}{\partial t^3} = 4u + \frac{\partial^2 u}{\partial x^2} - 8\cos 2t - 4\sin 2t. \tag{57}$$

with initial conditions:

xL/32	Exact solution for $t = 0.01$	Haar wavelet solution for $t = 0.01$	Absolute error for $t = 0.01$
1	9.797200996216327e-001	9.797031654960067e-001	1.693412562597807e-005
3	9.758943135988145e-001	9.758774455316912e-001	1.686806712331013e-005
5	9.682576811657250e-001	9.682409451074092e-001	1.673605831575831e-005
7	9.568400232085896e-001	9.568234844991506e-001	1.653870943907343e-005
9	9.416859254373909e-001	9.416696486624945e-001	1.627677489646207e-005
11	9.228545642796248e-001	9.228386129993005e-001	1.595128032427784e-005
13	9.004194757976174e-001	9.004039123012674e-001	1.556349635001286e-005
15	8.744682685317755e-001	8.744531535944705e-001	1.511493730499414e-005
17	8.451022813911115e-001	8.450876740364014e-001	1.460735471014996e-005
19	8.124361879269717e-001	8.124221451961642e-001	1.404273080751750e-005
21	7.765975485352725e-001	7.765841252649439e-001	1.342327032860524e-005
23	7.377263123358846e-001	7.377135609436286e-001	1.275139225598299e-005
25	6.959742706743144e-001	6.959622409537042e-001	1.202972061020713e-005
27	6.515044643797485e-001	6.514932033082534e-001	1.126107149507227e-005
29	6.044905470941043e-001	6.044800986361658e-001	1.044845793851668e-005
31	5.551161071582754e-001	5.551065121718027e-001	9.594986472682976e-006

Table 1. Comparison of Exact and Numerical Solutions of Example 1 at $t = 0.01$ for $J = 3$ and $\Delta t = 0.00001$.

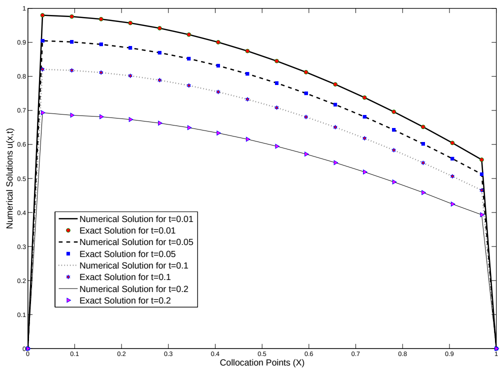

Figure 1. Comparison of Exact and Numerical Solutions of Example 1 at $t = 0.01, 0.05, 0.1, 0.2$ with $\Delta t = 0.00001$ and $J = 3$.

$$u(x,0) = e^{-x};\ \frac{\partial u}{\partial t}(x,0) = 2;\ \text{and}\ \frac{\partial^2 u}{\partial t^2}(x,0) = 0.$$

and boundary conditions:

$$u(0,t) = 1 + \sin 2t;\ u(1,t) = \cos 2 + \sin 2t.$$

Rewrite (57) in the operator form as:

$$L_t(u) = u_{xx} + 4u - 8\cos 2t - 4\sin 2t. \tag{58}$$

Applying inverse operator in above equation and developing the equation, we obtain:

$$u(x,t) = u(x,0) + t u_t(x,0) + \frac{t^2}{2} u_{tt}(x,0) + L_t^{-1}(u_{xx} + 4u - 8\cos 2t - 4\sin 2t). \tag{59}$$

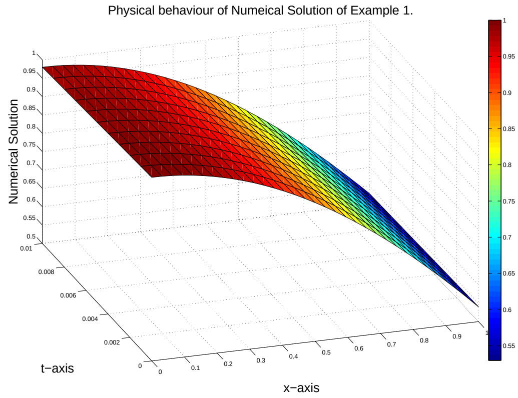

Figure 2. Physical behaviour of Numerical Solution for Example 1 at t=0.01 with $\Delta t = 0.001$ and $J = 3$.

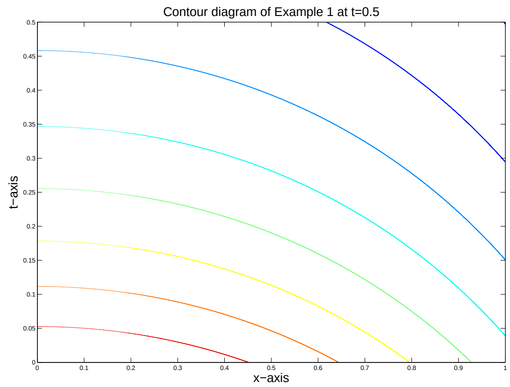

Figure 3. Contour diagram of Numerical Solutions of Example 1 at $t = 0.5$ with $\Delta t = 0.00001$ and $J = 3$.

For solving in t directions, use the following relations:

$$u_0 = u(x,0) + tu_t(x,0) + \frac{t^2}{2}u_{tt}(x,0) - L_t^{-1}(8\cos 2t + 4\sin 2t), \tag{60}$$

and

$$u_{k+1}(x,t) = L_t^{-1}(u_{xx} + 4u), \quad k \geqslant 0. \tag{61}$$

From here, components $u_0, u_1, u_2, ...$ are calculated as:

$$u_0 = \cos 2x + \sin 2t - \frac{\cos 2t}{2}, \tag{62}$$

$$u_1 = \frac{\cos 2t}{2} + \frac{\sin 2t}{4}, \tag{63}$$

$$u_2 = -\frac{\sin 2t}{4} + \frac{\cos 2t}{8}, \tag{64}$$

and so on. Finally, the series solution is obtained from (10). The solution obtained by Adomain decomposition method is:

$$u(x,t) = \cos 2x + \sin 2t. \tag{65}$$

The comparison of exact and numerical solutions by using Haar wavelet method for Example 2, is presented in Table 2 for $t = 0.01$ and $J = 3$ with $\Delta t = 0.00001$. Comparison of numerical solutions obtained by Haar wavelet method and Adomain decomposition method at different t are shown in Figure 2.

xL/32	Exact solution for $t = 0.01$	Haar wavelet solution for $t = 0.01$	Absolute error for $t = 0.01$
1	1.018046177393432e-000	1.017441925396709e-000	6.042519967228710e-004
3	1.002471979794589e-000	1.001867727797857e-000	6.042519967319748e-004
5	9.715666147415053e-001	9.709623627447704e-001	6.042519967348614e-004
7	9.258123501192694e-001	9.252080981225347e-001	6.042519967347504e-004
9	8.659231659244010e-001	8.653189139276661e-001	6.042519967348614e-004
11	7.928336128458046e-001	7.922293608490697e-001	6.042519967348614e-004
13	7.076842289138379e-001	7.070799769171031e-001	6.042519967347504e-004
15	6.118037417858105e-001	6.111994897890757e-001	6.042519967348614e-004
17	5.066883343952964e-001	5.060840823985610e-001	6.042519967354165e-004
19	3.939782975178662e-001	3.933740455211304e-001	6.042519967358051e-004
21	2.754324335821448e-001	2.748281815854090e-001	6.042519967357496e-004
23	1.529006111461583e-001	1.522963591494220e-001	6.042519967363325e-004
25	2.829489831719146e-002	2.769064632045468e-002	6.042519967367765e-004
27	-9.644027443151919e-002	-9.704452642825646e-002	6.042519967372761e-004
29	-2.193584564479885e-001	-2.199627084447248e-001	6.042519967363047e-004
31	-3.385415506128998e-001	-3.391458026096351e-001	6.042519967353055e-004

Table 2. Comparison of Exact and Numerical Solutions of Example 2 at $t = 0.01$ for $J = 3$ and $\Delta t = 0.00001$.

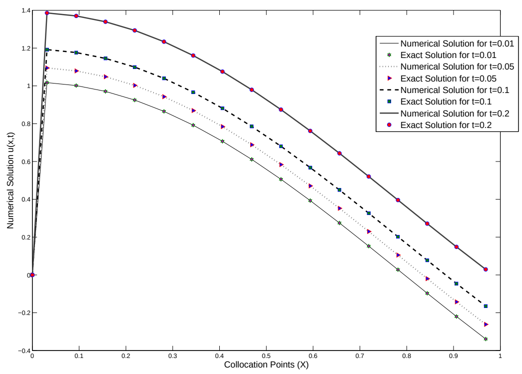

Figure 4. Comparison of Exact and Numerical Solutions of Example 2 at $t = 0.01, 0.05, 0.1, 0.2$ with $\Delta t = 0.00001$ and $J = 3$.

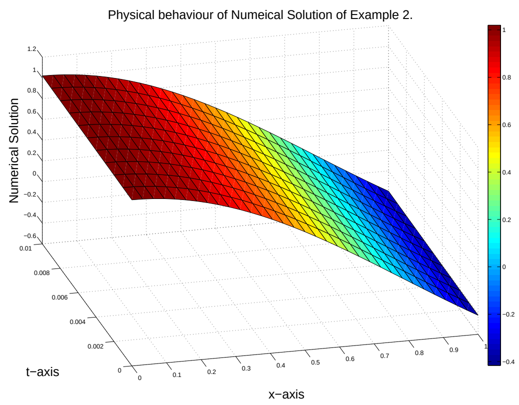

Figure 5. Physical behaviour of Numerical Solution for Example 2 at t=0.01 with $\Delta t = 0.001$ and $J = 3$.

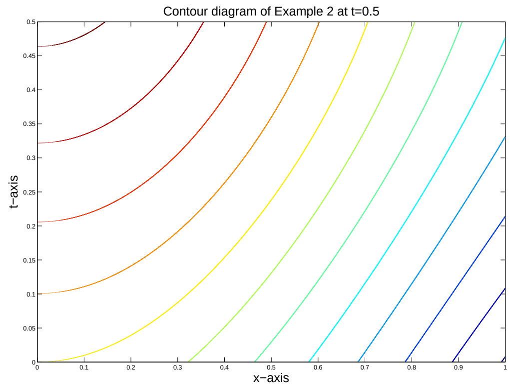

Figure 6. Contour diagram of Numerical Solutions of Example 2 at $t = 0.5$ with $\Delta t = 0.00001$ and $J = 3$.

7. Conclusion

We conclude here from the above, that the Haar wavelet method is more accurate, simple, fast and computationally efficient for solving third order partial differential equations. For getting the necessary accuracy the number of calculation points may be increased.

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