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Application of The Random Matrix Theory on the Cross-Correlation of Stock Prices

F. Sotoude Vanoliya[a,*] and A. Pourdarvish Heydari[b]

[a,b]*Department of Statistics, University of Mazandaran, Iran.*

Abstract. The analysis of cross-correlations is extensively applied for understanding of interconnections in stock markets. Variety of methods are used in order to search stock cross-correlations including the Random Matrix Theory (RMT), the Principal Component Analysis (PCA) and the Hierachical Structures. In this work, we analyze cross-crrelations between price fluctuations of 20 company stocks of Iran by using RMT. We find the eigenvalues and eigenvectors of the matrices of the cross-correlations related to these stocks. The results show some eigenvalues do not fall within the bounds of RMT eigenvalues, that indicate the correlations of stocks in usual and critical flucatutions.

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1. Introduction

In recent years, statistical characterizations of financial markets based on the concepts and the methods of physics have attracted considerable attention [2,3]. The stock market is a typical complex system with interactions between individuals, groups, and institutions at different levels [13]. At the same time studying the collapse of many financial markets, especially during a global recession, is of great importance. In turbulent stocks situation (crisis), the market show more volatility than calm and stable market. Many models have been proposed by both economists and physicists in order to explain the correlation of international financial markets , which is considered a complex system with many relations which are difficult to identify and quantify [3,4]. One tool that was first developed in nuclear phyisics for

*Corresponding author. Email: fereshte_sotoude@yahoo.com

studying complex systems with unknown correlation structure is random matrix theory [14].

Random-matrix theory gained attention during the 1950s due to work by Eugene Wigner in mathematical physics. Specialcally, Wigner wished to describe the general properties of the energy levels (or of their spacings) of highly excited states of heavy nuclei as measured in nuclear reactions. Such a complex nuclear system is represented by an Hermitian operator H (called the Hamiltonian) living in an infinite-dimensional Hilbert space governed by physical laws. Unfortunately, in any specific case, H is unknown. Moreover, even if it were known, it would be much complicated to write down and, even if we could write it down, no computer would be able to solve its eigenequation $Hv = \lambda v$ (the so-called Schrodinger equation of the physical system), where λ and v are an eigenvalue-eigenvector pair corresponding to H [5]. Despite this interpretation, a simple guess raised by Wigner. Under the principles of quantum mechanics, atomic nuclei, are like the steps of a ladder of high levels energy. To calculate the distance between these small steps, we first need to understand ways that one core can jump from one step to another, and then calculated the probability of each jump. Wigner did not know this and instead of that selected numbers to represent the probability and categorized them in an array of square matrix. The matrix was an easy way to express mutual relations between the stairs and the Wigner able to use these powerful mathematical tool to predict the level of nuclear energy [8]. Wigner argued that we should instead regard a speciafic Hamiltonian H as to behave like a large random matrix that is a member of a large class (or ensemble) of Hamiltonians, all of which would have similar general properties as the H in question. The energy levels (represented by the eigenvalues of H) of the physical system could then be approximated by the eigenvalues of a large random matrix. Furthermore, the spacings between energy levels of heavy nuclei could be more easily modelled by the spacings between successive eigenvalues of a random $n \times n$-matrix as $n \longrightarrow \infty$. Since 1960s, Wigner and his colleagues, including Freeman Dyson and Madan Lal Mehta, worked on random-matrix theory and developed it to the point that it became a very powerful tool in mathematical physics. The random matrix theory, originally developed in complex quantum system, was applied to analyze the cross-correlations between stocks in the U.S. stock market by Plerou et al. [5] . The statistics of the eigenvalues of the correlation matrix calculated from stock return series agree with the predictions of random matrix theory, but with deviations for few of larger eigenvalues. Extended work has been conducted to explain information contained in the deviating eigenvalues, which reveals that the largest eigenvalue corresponds to a marketwide influence to all stocks and the remaining deviating eigenvalues correspond to conventionally identified business sectors [11,12]. Using random matrix theory, many studies have been conducted on the correlation between different stocks [1, 6, 7, 9, 10, 15, 16, 17, 18, 19]. In recent years, there are increasing works concentrated on the variation of the cross-correlations between market equities over time . Aste et al. have investigated the evolution of the correlation structure among 395 stocks quoted on the U.S. equity market from 1996 to 2009, in which the connected links among stocks are built by a topologically constrained graph approach . They found that the stocks have increased correlations in the period of larger market instabilities[2]. During the last decade or so, we have seen more interest paid to random matrix theory. One of important early discoveries in random-matrix theory was its connection to quantum chaos, which leds to theory of quantum transport. Random-matrix theory has since become a major tool in many fields, including number theory and combinatorics, wireless communications, and in multivariate statistical analysis and principal components analysis. A common element in these types of situations is that random-matrix theory has been used as an indirect method for solving complicated problems arising from physical or mathematical systems [5].

2. Data Analysis

We analyzed the daily closing prices of 20 financial markets of the Tehran stock exchange from the priod June 2009 to June 2015. The financial indices are as follows: Iran.Tele.Co., Sobhan.Pharm, Calcimine, I.N.C.Ind., Iran.Khodro, Iran.Transfo, Bahman.Group, DPI, MAPNA, Mellat.Bank, Metals.&Min., Parsian.Bank, Behshahr.Ind., Pars.Minoo, Saderat.Bank, F.&.Kh.Cement, Gharb.Cement., Tehran.Cement, Behshahr.Inv. and Saipa. The data was collected from [20] and were divided into three periods.

The financial crisis of 2007-2009 as the worst financial crisis since the Great Depression of the 1930s originated from America and spread around the world. Therefore, 2009 is the year that still effects of the global financial crisis, It was evident that in 2010 and 2012 show generally mild bullish behavior so as in 2013 and 2015 due to political, economic and Some government decisions led to descending behavior into downward cycle. In order to make the cross-correlation matrix, in holidays the pervious day's closing price were taken in the matrix.

3. Random Matrix Theory Approach

Let $P_i(t)$ be the daily closing price of indices with $i = 1, ..., N$ where N is the total number of indices and the time spans $t = 1, ..., T$, where T is the maximum time of each window. The rate of change of price at time t is given by

$$R_i(t) = lnP_i(t+1) - lnP_i(t) \approx \frac{P_i(t+1) - P_i(t)}{P_i(t+1)}$$

Because of different stocks varying levels of volatility (standard deviation), we define a normalized return:

$$r_i(t) = \frac{R_i(t) - E(R_i(t))}{\sigma_i}$$

where σ_i is the standard deviation of R_i. The cross-correlation matrix C is expressed in terms of $r_i(t)$ as

$$C_{ij} = E[r_i(t)r_j(t)]$$

where C is a real, symmetric matrix with C_{ii} and C_{ij} has values in the range [-1,1]. Then, we compare the properties of C with those of a random cross-correlation matrix (Wishart matrix). The statistical properties of random matrices are known. Especially, as $N \longrightarrow \infty$ and $L \longrightarrow \infty$ with $Q = \frac{L}{N}(\geqslant 1)$ for N time series and L random elements with zero mean and unit variance, the probability density function of the eigenvalues λ of the random correlation matrix is given by

$$P_{rm}(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda} \quad (1)$$

for λ within the bounds $[\lambda_-, \lambda_+]$,where λ_- and λ_+ are the minimum and the maximum eigenvalues of random matrix, respectively, given by

$$\lambda_- = 1 + \frac{1}{Q} - 2\sqrt{\frac{1}{Q}} \tag{2}$$

$$\lambda_+ = 1 + \frac{1}{Q} + 2\sqrt{\frac{1}{Q}} \tag{3}$$

In addition, we discussed the effects of the global financial and economic crisis on Iran indicators. Figure 1 shows the volatilities (standard deviations) of three time windows in the stock market.

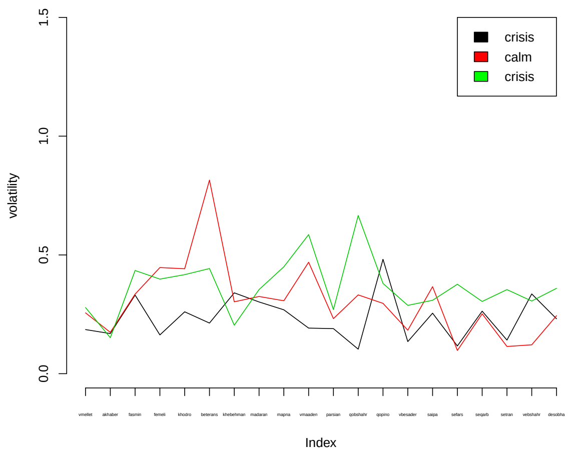

Figure 1. Volatility as a measure of fluctuations

It is obvious, the economic crisis in 2014 and 2015 show more fluctuations than two other periods, which represents the depth of crisis and the recession. We have calculated the cross-correlation matrix of the price changes. Figure 2 represents the probability distributions for the components of cross-correlation matrix of the stock market. The average values of the cross-correlation coefficients respectively are 0.1034207 the frist crisis, 0.1139561the calm period , and 0.1896877 the third period in the stock indices. The average cross correlation coefficient during the crisis period is higher than those two periods. The standard deviations of the cross-correlation coefficients are 0.2161215 the frist crisis, 0.2231051 the calm period and 0.2235 after the second crisis in the stock indices. The cross-correlation coefficients distributed broadly during the crisis periods. After the crisis (the calm period) the distribution becomes narrower.

The cross-correlation matrix of strongly correlated stock indices shows structure very different from that of a random matrix theory. For a real market some eigenvalues deviate from RMT predictions, which has been confirmed by several studies. In random matrix theory, the eigenvalues are bounded on the range $\lambda_- \leqslant \lambda \leqslant \lambda_+$, where the lower and the upper bounds of the eigenvalue are given by 2 and 3. We has Q : 18.9 24.05 and 29.8 respectively. The maximum (minimum) eigenvalues are $\lambda_{min(max)}$= 0.5928663 (1.449404) before the crisis,$\lambda_{min(max)}$= 0.6337563 (1.512954) during the crisis, and $\lambda_{min(max)}$= 0.6671854 (1.399929) after the crisis.

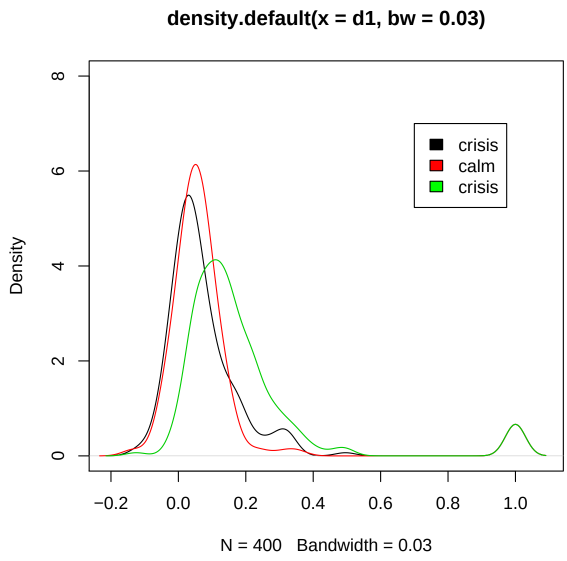

Figure 2. Probability density of the cross-correlation matrix

This indicates that for an uncorrelated time series, the eigenvalues should be bounded between the smallest and the largest eidenvalues. The eigenvalues of the empirical matrix for all periods deviate from RMT predictions in the market. The maximum (minimum) eigenvalues are $\lambda_{min(max)} = 0.5297632(2.2848594)$ for the first crisis , $\lambda_{min(max)} = 0.4113854(2.8597569)$ for the calm, and $\lambda_{min(max)} = 0.4361022(4.1807480)$ for the second crisis. The larger eigenvalue during the crisis shows that there is a stronger correlation among financial indices during the crisis. Figures 3 , 4 and 5 compares the probability distributions of eigenvalues for all periods.

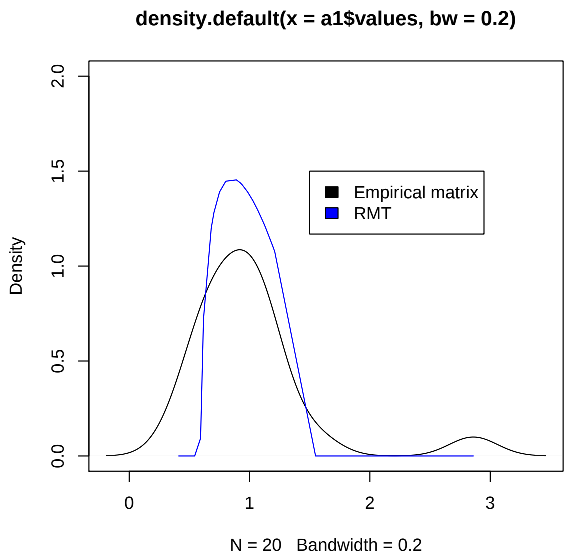

Figure 3. Probability density of eigenvalues for the frist period

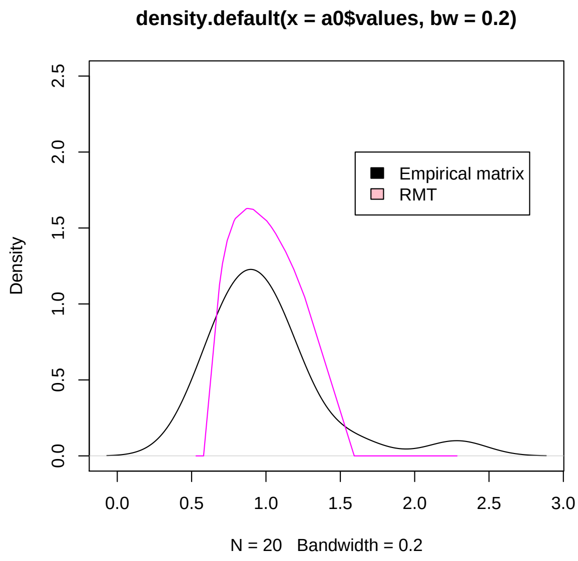

Figure 4. Probability density of eigenvalues for the second period

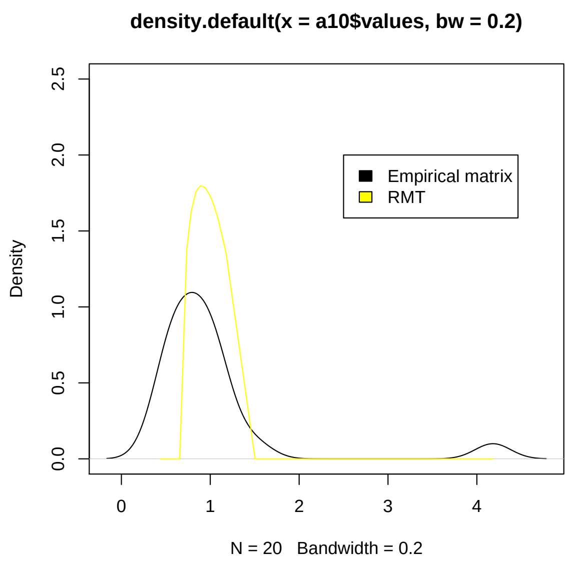

Figure 5. Probability density of eigenvalues for the third period

The eigenvectors corresponding to the largest eigenvalue are shown in Fig. 6. All the components carry the same sign which represents the same market mode, and there is no significant difference due to the crisis.

The components of the eigenvectors corresponding to the second largest eigenvalue are shown in Fig. 7 .

We find that most of components of the second period which carry negetive sign, switch to opposite directions in the third period. Generally, the indices which show large volatility (during the crisis) move into opposite direction during the crisis. The eigenvectors corresponding to the eigenvalues near to RMT predictions do not show any significant behavior. Less changes can be seen in first period than to second period.

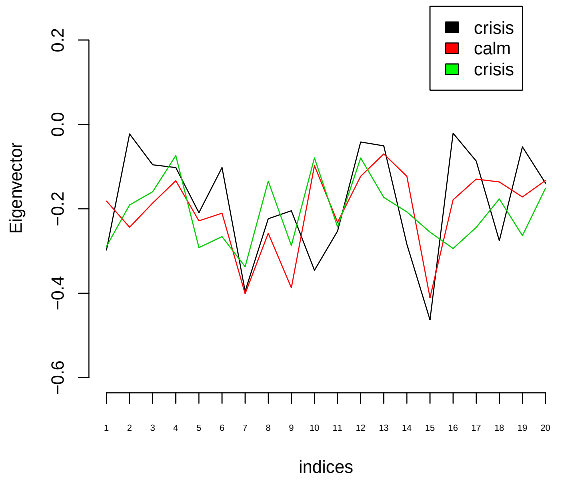

Figure 6. comparison of the omponents for the largest eigenvectors

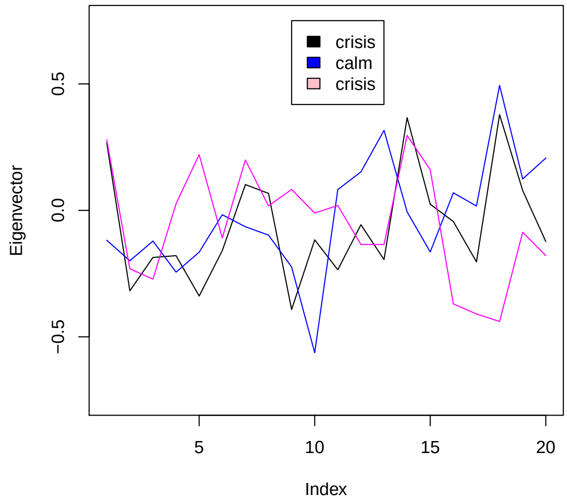

Figure 7. comparison of the omponents for the second largest eigenvectors

3.1 *The Inverse Participation Ratio*

The inverse participation ratio (IPR) of the eigenvector u^k is defined as

$$I^k = \sum_{l=1}^{N} (u_l^k)^4$$

, where u_l^k, $l = 1, \dots, N$, are the components of eigenvector u^k . A comparison of the IPR is shown in Fig.8.

The IPR quantifies the reciprocal of the number of eigenvector components that

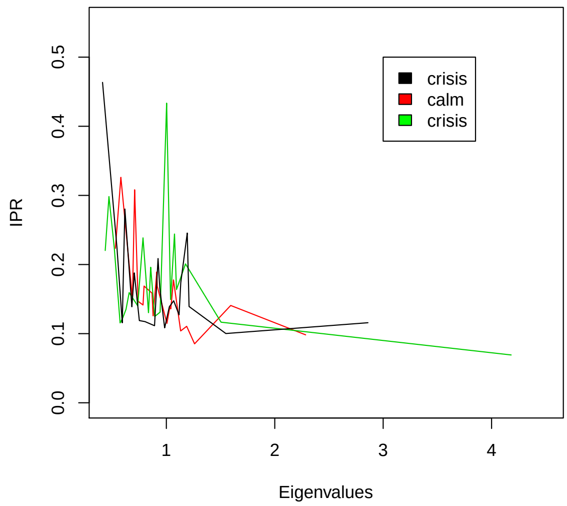

Figure 8. comparison of the IPR

contribute significantly. The largest value of $\frac{1}{I^k}$ is 11.72614 during the first crisis, 9.9815 in the seconed time, and 14.45874 the last time. This indicates that, during the crisis, more stocks participate in the largest eigenvector respeectively the seconed, the thired and the first eigenvector. In addition, we observe that the largest IPR respectively is 0.3261473, 9.9815, 0.4334882 which indicates that comparatively few stocks participate on the smallest eigenvector after the first crisis.

4. Counclution

We analyzed the cross-correlation matrices of stock price changes in stock indices for some years. We calculated the eigenvalues and eigenvectors of the correlation matrix. Almost all eigenvalues were in the predicted range of random matrix theory. However, some eigenvalues deviated from the predictions of random matrix theory for the indices . We observed that eigenvalues during the frist crisis were higher than they were during other periods. Then, we investigated the components of the largest and the second largest eigenvectors. We observed that the components of the second largest eigenvectors for the indices showed the same behaviors for all the periods , they showed opposite behaviors. We also observed more stock indices participate together by using the inverse participation ratio (IPR).

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