A Bi-level Formulation for Centralized Resource Allocation DEA Models

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Abstract. In this paper, the common centralized DEA models are extended to the bi-level centralized resource allocation (CRA) models based on revenue efficiency. Based on the Karush–Kuhn–Tucker (KKT) conditions, the bi-level CRA model is reduced to a one-level mathematical program subject to complementarity constraints (MPCC). A recurrent neural network is developed for solving this one-level mathematical programming problem. Under a proper assumption and utilizing a suitable Lyapunov function, it is shown that the proposed neural network is Lyapunov stable and convergent to an exact optimal solution of the original problem. Finally, an illustrative example is elaborated to substantiate the applicability and effectiveness of the proposed approach.

1. Introduction

Data envelopment analysis (DEA), originally developed by Charnes et al. [5], is a linear programming methodology for assessing relative efficiency and productivity of multiple inputs and outputs decision making units (DMUs). In its recent developments, DEA has had more of a planning orientation for resource allocation problems. The use of DEA provides an alternative to the resource allocation problem because it is possible in this method to consider feasible production plans and trade-offs between the inputs/outputs based on the empirical characterization of a production possibility set [13]. The main limitation of a traditional DEA model in resource allocation is that it analyzes one unit at a time independently. Many authors have developed a number of centralized DEA-based models from different perspectives [1,2,8,10-12,15]. However, most papers in the literature have proposed centralized resource allocation models from the technical efficiency perspective.

Since the computing time needed to solve a DEA problem greatly depends on its dimension and structure, traditional algorithms cannot evaluate the efficiency of large

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scale data sets. In general, the large-scale DEA problems with negative data are challenging problems. Unlike traditional algorithms, artificial neural networks have massively paralleled distributed computation, fast convergence and robust solution. Therefore, artificial neural networks can be considered as a promising approach to solve the large-scale DEA problem in real time.

Neural networks for solving mathematical programming problems were first introduced in the 1980s by Hopfield and Tank [16]. The main feature of these neural networks is that its equilibrium point coincides with the solution of the underlying optimization problem.

Motivated by the above discussions, in this paper, we extend the common centralized DEA models to the bi-level CRA models based on revenue efficiency and a cost analysis across a set of DMUs under a centralized decision-making environment. Also, we present a recurrent neural network for solving the proposed bi-level CRA model. This neural network is proved to be globally stable by constructing a suitable Lyapunov function and the solution trajectory can converge to an optimal solution of the original optimization problem.

The rest of the paper is organized as follows. In Section 2, we introduce the traditional revenue efficiency DEA formulations. In Section 3, we develop a bi-level centralized DEA model based on revenue efficiency for resource reallocation also present selected extensions to the model. In Section 4, a neural network model is designed for solving the proposed bi-level CRA model and its stability is analyzed. An empirical example is given in Section 5. Finally, some conclusions are drawn in Section 6.

2. Preliminaries

Before formulating the models, the required notation needs to be introduced. Let

- $n$ Number of observed units
- $x_j$ Input vector of DMU $j$
- $y_j$ Output vector of DMU $j$
- $p$ Price vector of outputs
- $p_{rj}$ Price of output $r$ of unit $j$
- $\lambda_j$ Intensity variable for DMU $j$

2.1. Traditional revenue efficiency model

We assume that there are $n$ DMUs and that each DMU uses $m$ inputs to produce $s$ outputs. For each DMU $j$ ($j = 1, \ldots, n$), we denote the input and the output vectors as $(x_j, y_j)$, where $x_j = (x_{1j}, \ldots, x_{mj})$ and $y_j = (y_{1j}, \ldots, y_{sj})$. Arranging the data set in matrices $X = (x_j)$ and $Y = (y_j)$, and assuming that $X > 0$ and $Y > 0$, the production possibility set under a Variable Returns to Scale (VRS) technology is generally denoted as follows:

$$T = \left\{(x, y) \mid x \succeq \sum_{j=1}^{n} x_j \lambda_j, \ y \preceq \sum_{j=1}^{n} y_j \lambda_j, \ \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0, \ j = 1, \ldots, n \right\}.$$

The standard DEA model based on this assumption is called the BCC model [3]. Given the common unit price vector $p = (p_1, \ldots, p_n)$ for the output $y$, the maximum revenue
for DMU_a is obtained as follows [12,13]:
\[
\begin{align*}
\max & \quad \mathbf{p}^T \mathbf{y} + \varepsilon \mathbf{1}^T \mathbf{s}^- \\
\text{s.t.} & \quad \mathbf{X} \lambda + \mathbf{s}^{-} = \mathbf{x}_a, \\
& \quad \mathbf{Y} \lambda = \mathbf{y}, \\
& \quad \mathbf{1}^T \lambda = 1, \\
& \quad \mathbf{s}^{-} \geq 0, \quad \lambda \geq 0,
\end{align*}
\]
(1)
where \( \mathbf{1} = [1, \ldots, 1]^T \in \mathbb{R}^n \) and \( \varepsilon \) represents a non-Archimedean infinitesimal. The slack vector \( \mathbf{s}^{-} = (s_1^{-}, s_2^{-}, \ldots, s_m^{-}) \in \mathbb{R}^m \) represents the actual inputs that can be reduced. \( \lambda = (\lambda_1, \ldots, \lambda_s)^T \) is a non-negative vector in \( \mathbb{R}^s \), and \( \mathbf{y} \) is the revenue maximizing level produced by DMU_a given the output price \( p \) and the input level \( \mathbf{x}_a \). Let the optimal solution of (1) be \( (\mathbf{y}^*, \lambda^*) \). Then, the revenue efficiency of DMU_a is defined as the ratio of maximum revenue to observed revenue via the following formula:
\[
RE_a \text{(Revenue Efficiency)} = \frac{p^T \mathbf{y}^*}{p^T \mathbf{y}_a}.
\]

2.2 Resource reallocation based on revenue efficiency

We assume that all of the units operate under a central unit with the control of some decision parameters, such as resources of those units. To allocate the input resources to a set of existing units so that the total output revenue will be maximized, Fang and Li [8] proposed the revenue allocation model based on revenue efficiency within the original production possibility set as follows:
\[
\begin{align*}
\max & \quad \varphi = \sum_{i=1}^{n} \sum_{r=1}^{k} p_{ir} \overline{y}_{ik} + \varepsilon \sum_{m=1}^{n} \sum_{i=1}^{k} s_{ik}^- \\
\text{s.t.} & \quad \sum_{j=1}^{n} \lambda_{jk} x_{kj} \leq x_{jk}, \quad k = 1, \ldots, n, \quad i \in U, \\
& \quad \sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_{ik}^- = \overline{x}_{ik}, \quad k = 1, \ldots, n, \quad i \not\in U, \\
& \quad \sum_{j=1}^{n} \lambda_{jk} y_{ij} = \overline{y}_{ik}, \quad k = 1, \ldots, n, \quad r = 1, \ldots, s, \\
& \quad \sum_{i=1}^{n} \overline{x}_{ik} = \sum_{i=1}^{n} x_{ikt}, \quad i = 1, \ldots, m, \quad i \not\in U, \\
& \quad \sum_{j=1}^{n} \lambda_{jk} = 1, \quad k = 1, \ldots, n, \\
& \quad \lambda_{jk} \geq 0, \quad k = 1, \ldots, n, \quad j = 1, \ldots, n,
\end{align*}
\]
(2)
where \( j, k = 1, \ldots, n \) are the indexes for the DMUs, and \( U \) represents the set of un-reallocatable variables. \( \begin{pmatrix} \overline{x}_{ik} \\ \overline{y}_{ik} \end{pmatrix} \) describes the input-output target after the reallocation \( (\lambda_{ij}, \lambda_{2j}, \ldots, \lambda_{nj}) \) represents the vector for projecting the DMU_j. \( s_{ik}^- \) indicates the slack of input \( i \) for DMU_j. Also, \( \varepsilon \) is a non-Archimedean element smaller than any positive real number.

The reallocation of input resources among the DMUs may incur costs, which includes the transportation costs, miscellaneous labor costs, etc. Therefore, Fang and Li [8] developed the following linear programming problem in which the reallocation amounts are taken to their minimum cost:
min ξ = \sum_{i=1}^{n} \sum_{k=1}^{m} c_{ik}γ_{ik}^+ + \sum_{i=1}^{n} \sum_{k=1}^{m} c_{ik}γ_{ik}^-
\text{s.t.} \sum_{j=1}^{m} \lambda_{jk} x_{ij} \leq x_{ij}, \quad k = 1,\ldots,n, \quad i \in U,
\sum_{j=1}^{m} \lambda_{jk} x_{ij} + s_{ik}^+ = x_{ij}, \quad k = 1,\ldots,n, \quad i \notin U,
\sum_{j=1}^{m} \lambda_{jk} y_{ij} = \bar{y}_{ik}, \quad k = 1,\ldots,n, \quad r = 1,\ldots,s,
\sum_{k=1}^{m} \bar{x}_{ik} = \sum_{r=1}^{s} \bar{y}_{ik}, \quad i = 1,\ldots,m, \quad i \notin U,
\sum_{j=1}^{m} \lambda_{jk} = 1, \quad k = 1,\ldots,n,
\lambda_{jk} \geq 0, \quad k = 1,\ldots,n, \quad j = 1,\ldots,n.
(3)

The term (UP) is called the upper level problem and (LP) is called the lower level problem.

3. A bi-level formulation for CRA based on the cost-revenue analysis

In this section, we present a bi-level DEA-based model for centralized resource allocation based on revenue efficiency and a cost analysis across a set of decision making units (DMUs) under a centralized decision-making environment. The upper-level model is concerned with determining the minimum reallocation cost while input resources and output targets are evaluated in the lower-level model.

The bi-level programming problem (BPP) corresponding to the linear program (2) and (3) can be described as follows:

(UP) min ξ = \sum_{i=1}^{n} \sum_{k=1}^{m} c_{ik}γ_{ik}^+ + \sum_{i=1}^{n} \sum_{k=1}^{m} c_{ik}γ_{ik}^-
\text{s.t.} \bar{x}_{ik} + γ_{ik}^+ - γ_{ik}^- = x_{ij}, \quad k = 1,\ldots,n, \quad i \notin U,
γ_{ik}^- \geq 0, \quad γ_{ik}^+ \geq 0, \quad k = 1,\ldots,n, \quad i \notin U,
(4)

(LP) max φ= \sum_{i=1}^{n} \sum_{k=1}^{m} p_{ik} \bar{y}_{ik} + \epsilon \sum_{i=1}^{n} \sum_{k=1}^{m} s_{ik}^-
\text{s.t.} \sum_{j=1}^{m} \lambda_{jk} x_{ij} \leq x_{ij}, \quad k = 1,\ldots,n, \quad i \in U,
\sum_{j=1}^{m} \lambda_{jk} x_{ij} + s_{ik}^+ = x_{ij}, \quad k = 1,\ldots,n, \quad i \notin U,
At the UP, the decision maker has to choose first a vector \((v_a^+, v_a^-)^T, k = 1, \ldots, n, i \in U\), to minimize his objective function \(\bar{F}\); then under this decision the LP decision maker has to select the decision vector \((x_{ik}, y_{ik}, \lambda_{ik}, \xi_{ik})^T, k = 1, \ldots, n, j = 1, \ldots, n, i \notin U\), that minimizes his own objective \(\bar{\phi}\).

We can reduce the BPP (4) to the one-level programming problem by replacing the lower-level problem with its KKT optimality condition.

4. Neural network model

In this section, we introduce a recurrent neural network for solving (4). Let \(w \in W \subset \mathbb{R}^r, z \in Z \subset \mathbb{R}^h, F : W \times Z \to \mathbb{R}, f : W \times Z \to \mathbb{R}\). Without loss of generality, we consider the following bi-level programming problem:

\[
\begin{align*}
\text{(UP)} & \quad \min_{w \in W} F(w, z) = c_1^T w + d_1^T z, \\
\text{s.t.} & \quad A_1 w + B_1 z \leq b_1, \\
\text{(LP)} & \quad \min_{z \in Z} f(w, z) = c_2^T w + d_2^T z, \\
\text{s.t.} & \quad A_2 w + B_2 z \leq b_2, \\
& \quad c_1, c_2 \in \mathbb{R}^r, d_1, d_2 \in \mathbb{R}^h, b_1 \in \mathbb{R}^p, b_2 \in \mathbb{R}^q, \\
& \quad A_1 \in \mathbb{R}^{p \times r}, B_1 \in \mathbb{R}^{p \times h}, A_2 \in \mathbb{R}^{q \times r}, B_2 \in \mathbb{R}^{q \times h}.
\end{align*}
\] (5)

**Assumption 1.** The constraint region of the above BPP

\[S = \{(w, z): w \geq 0, z \geq 0, A_1 w + B_1 z \leq b_1, A_2 w + B_2 z \leq b_2\},\]

is nonempty and compact.

Following the above assumption, we can reduce the BPP (5) to the one-level programming problem:

\[
\begin{align*}
\min & \quad F(w, z) \\
\text{s.t.} & \quad A_1 w + B_1 z \leq b_1, \\
& \quad A_2 w + B_2 z \leq b_2, \\
& \quad d_1^T u - \nu = 0, \\
& \quad \nu^T (b_2 - A_2 w - B_2 z) = 0, \\
& \quad w \geq 0, z \geq 0, u \geq 0, \nu \geq 0,
\end{align*}
\] (6)

where \(u \in \mathbb{R}^p\) and \(\nu \in \mathbb{R}^q\). Problem (6) is non-convex and non-smooth problem, and it is not good for using the neural network approach to solve problem (6). But fortunately, smoothing method is also presented in [6,9] for MPCC (6). Following this smoothing method, we can propose a neural network approach for problem (5).

**Definition 4.1.** A function \(\phi : \mathbb{R}^2 \to \mathbb{R}\) is called an NCP function if it satisfies

\[\phi(a, b) = 0 \iff a \geq 0, b \geq 0, ab = 0.\]

A popular NCP-function is the perturbed Fischer-Burmeister (FB) function, which is defined as

\[\phi^{\epsilon}_{FB}(a, b) = \sqrt{a^2 + b^2 + \epsilon} - a - b, \quad \epsilon \to 0^+.\]

The important property of \(\phi^{\epsilon}_{FB}\) can be stated in the following result.

**Proposition 4.1.** ([9]) For every \(\epsilon \in \mathbb{R}\), we have...
\[ \phi_{tb}(a,b) = 0 \iff a > 0, \ b > 0, \ ab = \frac{e}{2}. \]

By the Proposition 4.1, MPCC (6) can be rewritten as follows:

\[
\begin{array}{c}
\text{min } \, F(w, z) \\
\text{s.t. } \quad A_iw + B_i z \leq b_i, \\
\quad d_z + B_{i}^{T} u - v = 0, \\
\quad \sqrt{u_i^2 + (b_i - A_i w - B_i z)^2} + \epsilon - u_i - (b_i - A_i w - B_i z), \quad i = 1, \ldots, q, \\
\quad \sqrt{v_j^2 + z_j^2} + \epsilon - v_j - z_j, \quad j = 1, \ldots, h, \\
\quad w \geq 0.
\end{array}
\]  \quad (7)

For notational convenience, introducing

\[ \bar{w} = (w, z, u, v), \]
\[ G(w, z, u, v) = \begin{pmatrix} A_i w + B_i z - b_i \\ -x \end{pmatrix}, \]
\[ H(w, z, u, v) = \begin{pmatrix} d_z + B_{i}^{T} u - v \\ \phi_{tb}^e(u_i, (b_i - A_i w - B_i z)), \quad i = 1, \ldots, q, \\ \phi_{tb}^e(v_j, z_j), \quad j = 1, \ldots, h \end{pmatrix}. \]

(7) can be rewritten as the following equivalent problem:

\[
\begin{array}{c}
\text{min } \, F(\bar{w}) = c_i^T w + d_{i}^T z \\
\text{s.t. } \quad G_i(\bar{w}) \leq 0, \quad l = 1, \ldots, t + p, \\
\quad H_{o}(\bar{w}) = 0, \quad o = 1, \ldots, 2h + q, \\
\end{array}
\]  \quad (8)

**Definition 4.2.** Let \( \bar{w} \) be a feasible point of problem (8) and \( L = \{ l : G_l(\bar{w}) = 0, \ l = 1, \ldots, t + p \} \). We say that \( \bar{w} \) is a regular point of problem (8) if the gradients \( \nabla H_{o}(\bar{w}), \ o = 1, \ldots, 2h + q \), and \( \nabla G_{l}(\bar{w}), \ l \in L \) are linearly independent.

**Theorem 4.1.** Let \( \{ \bar{w}^e \} \) be a sequence of solutions of (8). Suppose that the sequence \( \{ \bar{w}^e \} \) converges to some \( \bar{w} \) for \( \epsilon \to 0^+ \). If \( \bar{w} \) is a regular point, then \( \bar{w} \) solves the problem (5).

Now, consider the Lagrangian function associated with (8) as follows:

\[ L(\bar{w}, \mu, \alpha, \beta) = F(\bar{w}) + \sum_{i=1}^{2h+q} \alpha_i H_i(\bar{w}) + \sum_{i=1}^{2h+q} \beta_i [G_i(\bar{w}) + \mu_i^2], \]

where the term \( \mu \) is slack variable, and the terms \( \alpha, \beta \) are referred as Lagrange multipliers.

The aim is to design a continuous-time dynamical system for solving the problem (5). The proposed neural network model is described as follows:

\[
\begin{align*}
\frac{d\bar{w}}{dt} &= -\nabla_{\bar{w}} L(\bar{w}, \mu, \alpha, \beta), \\
\frac{d\mu}{dt} &= -\nabla_{\mu} L(\bar{w}, \mu, \alpha, \beta), \\
\frac{d\alpha}{dt} &= \nabla_{\alpha} L(\bar{w}, \mu, \alpha, \beta), \\
\frac{d\beta}{dt} &= \nabla_{\beta} L(\bar{w}, \mu, \alpha, \beta).
\end{align*}
\]  \quad (9)
Similarly, a recurrent neural network can be defined for solving the problem (4). Now, we study stability properties of the neural network whose dynamics is described by the nonlinear differential equations (9).

**Theorem 4.2.** Let \((\tilde{w}^*, \mu^*, \alpha^*, \beta^*)\) be the equilibrium of the neural network (9), and assume that \(\tilde{w}^*\) is a regular point of problem (8). Then the equilibrium of the neural network solves problem (8).

**Theorem 4.3.** Let \((\tilde{w}^*, \mu^*, \alpha^*, \beta^*)\) be the equilibrium of the neural network (9). If \(\tilde{w}^*\) is the regular point of problem (8), then \((\tilde{w}^*, \mu^*, \alpha^*, \beta^*)\) is an asymptotically stable point of the neural network.

### 5. Illustrative example

In order to demonstrate the effectiveness and efficiency of the proposed approach, in this section, we analyze an empirical data set that is extracted from real application.

The data set is extracted from a real life case, 20 fast-food restaurants located in the city of Hefei, AnHui Province, China, presented by Du et al. [7]. These fast-food restaurants belong to the same chain, which has a central unit with the authority to supervise the operations of all branches and allocate resources among them. Table 1 shows the input and output dataset for 20 fast-food restaurants.

<table>
<thead>
<tr>
<th>DMU</th>
<th>Inputs</th>
<th>outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Man-hour</td>
<td>Shop</td>
</tr>
<tr>
<td></td>
<td>10^3 h</td>
<td>Size</td>
</tr>
<tr>
<td>-----</td>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>3.20</td>
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</tr>
<tr>
<td>2</td>
<td>3.40</td>
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</tr>
<tr>
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</tr>
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<td>5</td>
<td>4.20</td>
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</tr>
<tr>
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<td>4.10</td>
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<td>9</td>
<td>2.90</td>
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<tr>
<td>10</td>
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<td>11</td>
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<td>3.60</td>
<td>1.90</td>
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<tr>
<td>20</td>
<td>3.10</td>
<td>1.70</td>
</tr>
<tr>
<td>Total</td>
<td>71.6</td>
<td>42.8</td>
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</table>
Five output variables and two input variables are considered. The term ‘man-hours’ refers to the labor force used within a certain period, and the shop size is the total rental floor space of the restaurant used for serving customers. The outputs are the sales of meat dishes, vegetables dishes, soup, noodles and beverages. The output price vector is $p = (32, 15, 10, 6, 3)^T$. The current total revenue is 2926.35.

Because these fast food restaurants belong to the same organization, a central decision-maker can simultaneously control all of the DMUs. Note that the input shop size is a non-reallocatable variable. We assume that the central decision maker can reallocate the man-hour resources among these 20 fast-food restaurants to maximize the total revenue. The results from model (2) and (3) are given in Table 2, where reallocation of current man-hour resources was allowed. The total revenue is 3105.36.

The results of bi-level model (4) and proposed neural network (9) are shown in Table 3 below in an example that allowed for current resources man-hour reallocation. The total revenue is 3109.12 and increases by $3109.12 / 2926.35 - 1 = 6.24\%$ compared with the original total revenue.

Table 2: Results of resources reallocation by Fang and Li [8].

<table>
<thead>
<tr>
<th>DMU</th>
<th>Man-hour $10^3 h$</th>
<th>Meat dish $10^3$ servings</th>
<th>Vegetable dish $10^3$ servings</th>
<th>Soup $10^3$ servings</th>
<th>Noodle $10^3$ servings</th>
<th>Beverage $10^3$ servings</th>
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<td>1.372</td>
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<td>1.408</td>
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<td>1.004</td>
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<td>2.68</td>
<td>2.582</td>
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Table 3: Results of resources reallocation for 20 fast-food restaurants using proposed model 4 and neural network (9).

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<th>DMU</th>
<th>Inputs 10^3 Man-hour</th>
<th>Outputs 10^3 Meat dish servings</th>
<th>Outputs 10^3 Vegetable dish servings</th>
<th>Outputs 10^3 Soup servings</th>
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6. Conclusion

In this paper, we extended the common centralized DEA models to the bi-level CRA models based on revenue efficiency and a cost analysis across a set of DMUs under a centralized decision-making environment. Also, a recurrent neural network has been designed for solving the proposed bi-level CRA model. Based on Lyapunov stability theory, the proposed neural network has been proved to be globally asymptotically stable and capable of generating exact optimal solution of the proposed bi-level CRA model. To further demonstrate the advanced features of our approach and its practical relevance, we analyzed an empirical data set that was extracted from real applications. By making comparisons of the proposed approach with the existing approaches, it has been seen that the proposed approach in this paper can achieve higher total revenue.

References